SMT@Microsoft

Midwest Verification Day, Iowa, 2009

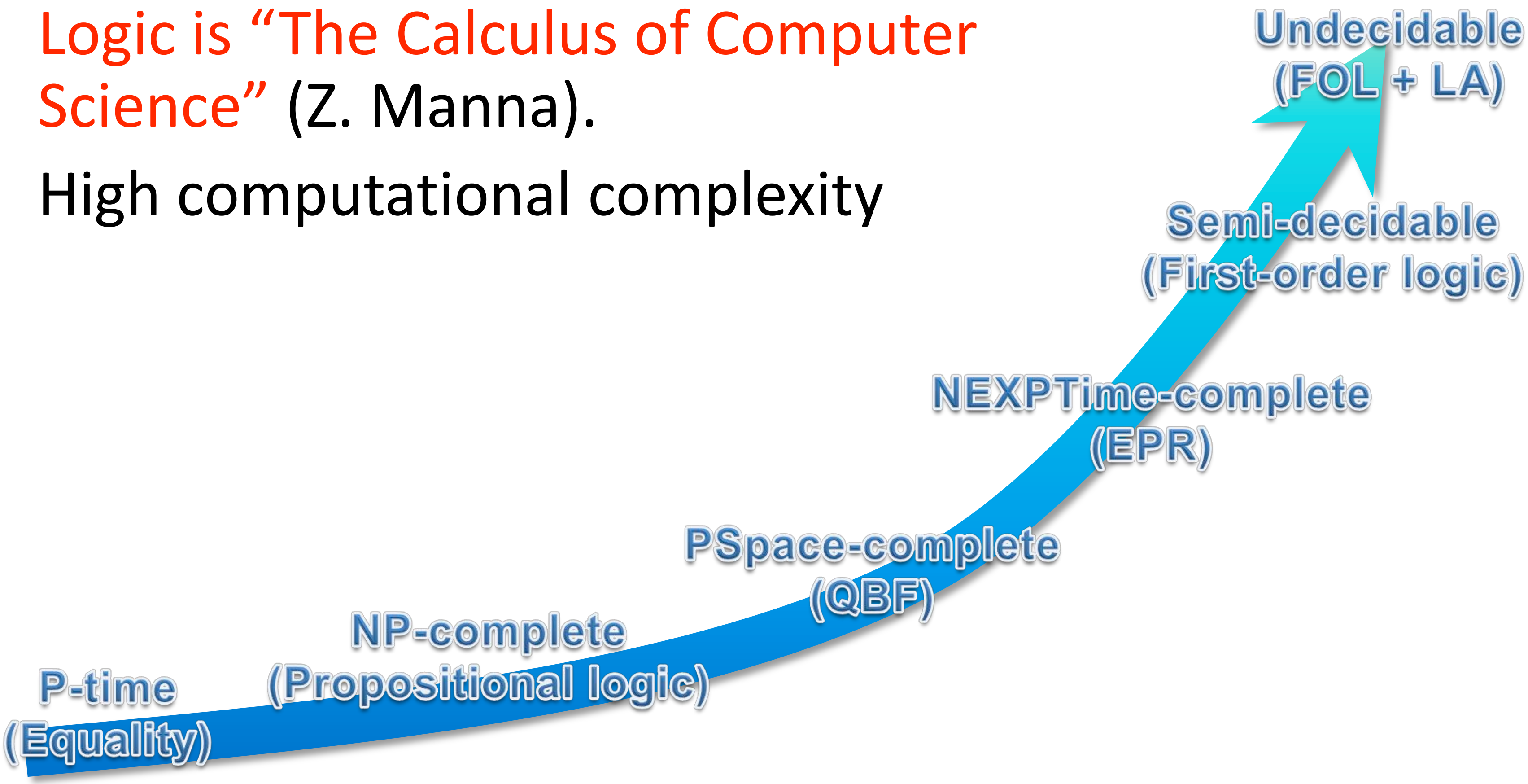
Leonardo de Moura
Microsoft Research

Symbolic Reasoning

Verification/Analysis tools need some form of **Symbolic Reasoning**

Symbolic Reasoning

- Logic is "The Calculus of Computer Science" (Z. Manna).
- High computational complexity



Applications

- Test case generation
- Verifying Compilers
- Predicate Abstraction
- Invariant Generation
- Type Checking
- Model Based Testing

Some Applications @ Microsoft

The Spec# Programming System

HAVOC

ForμLa

Hyper-V Microsoft Virtualization

Terminator T-2

VCC

SLAM

Yogi

NModel

Vigilante

SpecExplorer

Pex

F7

SAGE

Test case generation

```
unsigned GCD(x, y) {
  requires(y > 0);
  while (true) {
    unsigned m = x % y;
    if (m == 0) return y;
    x = y;
    y = m;
  }
}
```

SSA →

$(y_0 > 0)$ and

$(m_0 = x_0 \% y_0)$ and

not $(m_0 = 0)$ and

$(x_1 = y_0)$ and

$(y_1 = m_0)$ and

$(m_1 = x_1 \% y_1)$ and

$(m_1 = 0)$

Solver →

$x_0 = 2$

$y_0 = 4$

$m_0 = 2$

$x_1 = 4$

$y_1 = 2$

$m_1 = 0$

We want a trace where the loop is executed twice.

Type checking

Signature:

div : int, { x : int | x $\neq$ 0 } $\rightarrow$ int

Subtype

Call site:

if a $\leq$ 1 and a $\leq$ b then

return div(a, b)

Verification condition

a $\leq$ 1 and a $\leq$ b implies b $\neq$ 0

Satisfiability Modulo Theories (SMT)

Is formula *F* satisfiable modulo theory *T* ?

SMT solvers have specialized algorithms for *T*

Satisfiability Modulo Theories (SMT)

$$b + 2 = c \text{ and } f(read(write(a,b,3), c-2)) \neq f(c-b+1)$$

Satisfiability Modulo Theories (SMT)

$$\boxed{b + 2 = c} \text{ and } f(read(write(a,b,\boxed{3}), \boxed{c-2}) \neq f(\boxed{c-b+1})$$

Arithmetic

Satisfiability Modulo Theories (SMT)

$b + 2 = c$ and $f(\boxed{read}(\boxed{write}(a,b,3), c-2)) \neq f(c-b+1)$

Array Theory

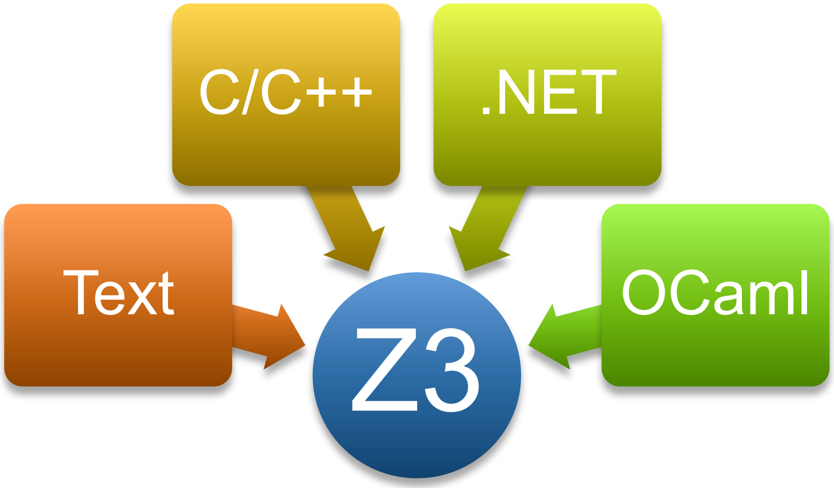
Satisfiability Modulo Theories (SMT)

$$b + 2 = c \text{ and } \boxed{f}(read(write(a,b,3), c-2)) \neq \boxed{f}(c-b+1)$$

Uninterpreted Functions

SMT@Microsoft: Solver

- Z3 is a new solver developed at Microsoft Research.
- Development/Research driven by internal customers.
- Free for academic research.
- Interfaces:



- http://research.microsoft.com/projects/z3

Ground formulas

For most SMT solvers: ***F is a set of ground formulas***

Many Applications

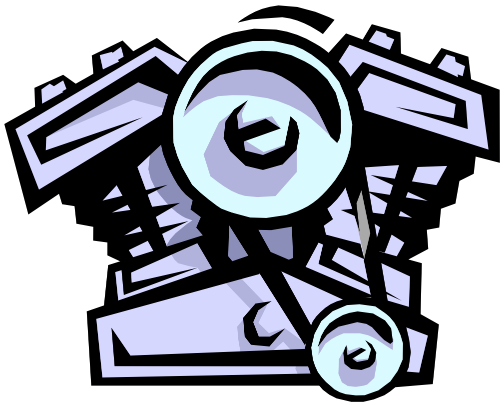
Bounded Model Checking

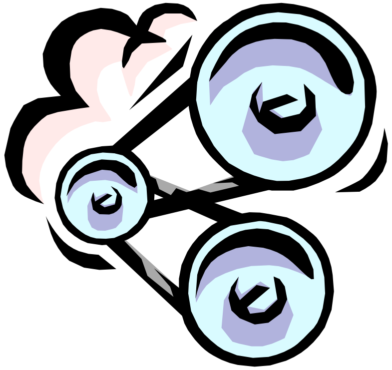
Test-Case Generation

Little Engines of Proof

An SMT Solver is a collection of

Little Engines of Proof





Deciding Equality

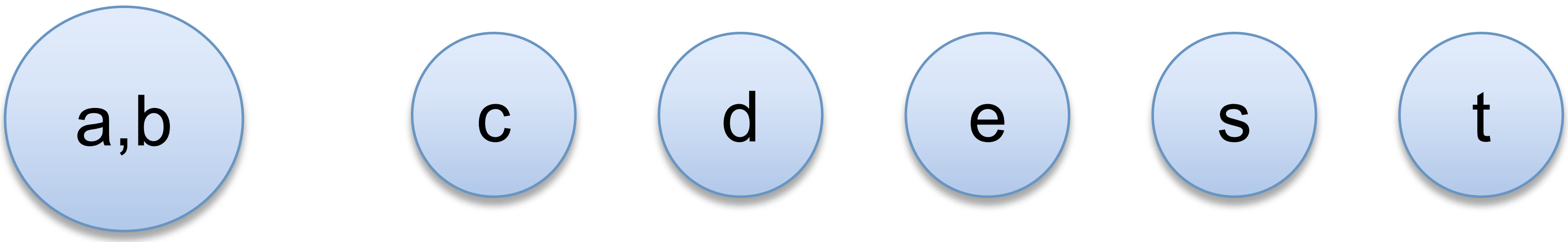
$$a = b,\ b = c,\ d = e,\ b = s,\ d = t,\ a \neq e,\ a \neq s$$

Deciding Equality

$a = b$, $b = c$, $d = e$, $b = s$, $d = t$, $a \neq e$, $a \neq s$

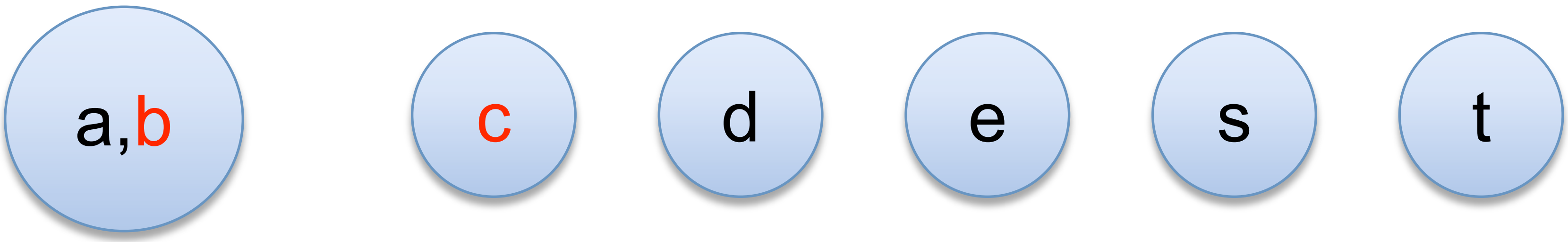
Deciding Equality

$\color{red}{a = b}, b = c, d = e, b = s, d = t, a \neq e, a \neq s$



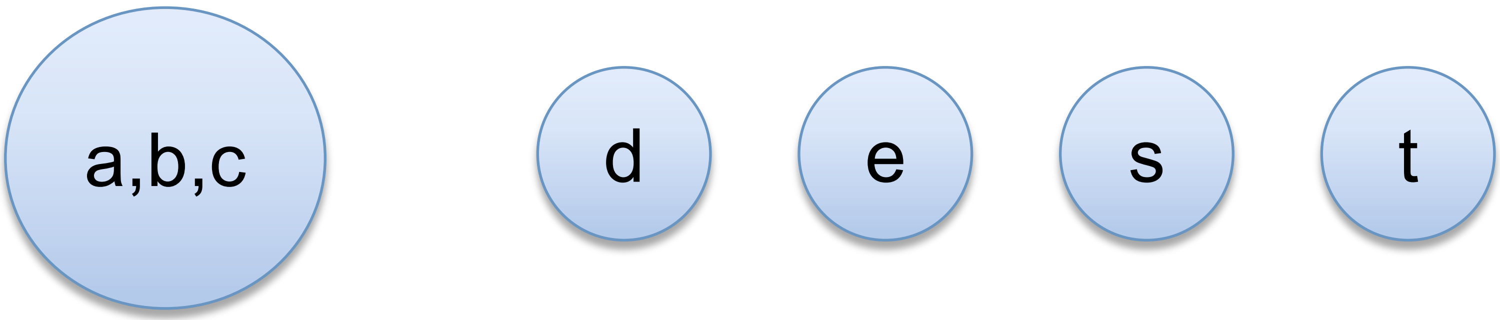
Deciding Equality

$a = b$, $b = c$, $d = e$, $b = s$, $d = t$, $a \neq e$, $a \neq s$



Deciding Equality

$$a = b,\ {\color{red} b = c},\ d = e,\ b = s,\ d = t,\ a \neq e,\ a \neq s$$



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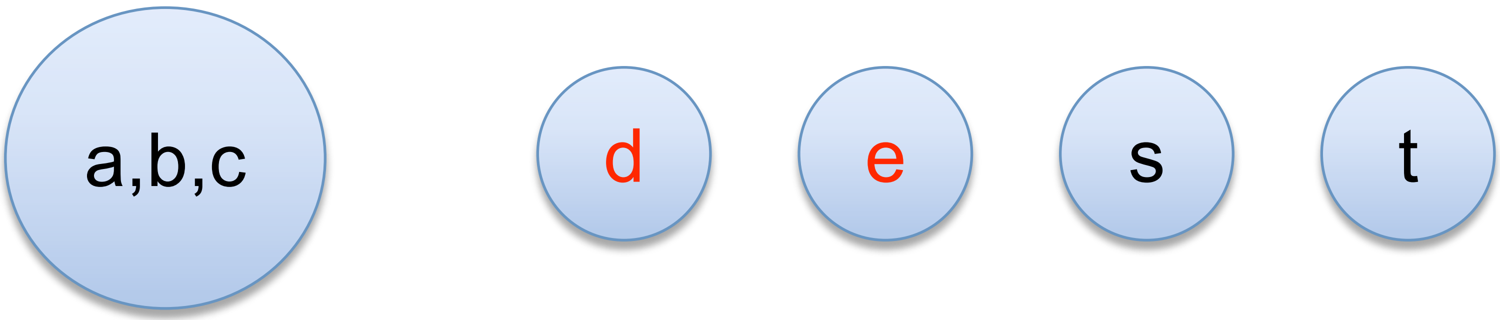

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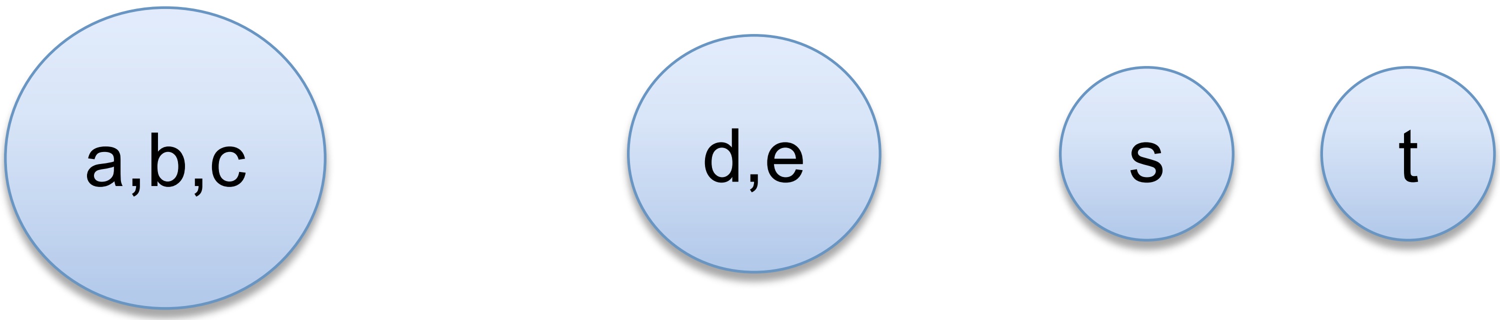

Deciding Equality

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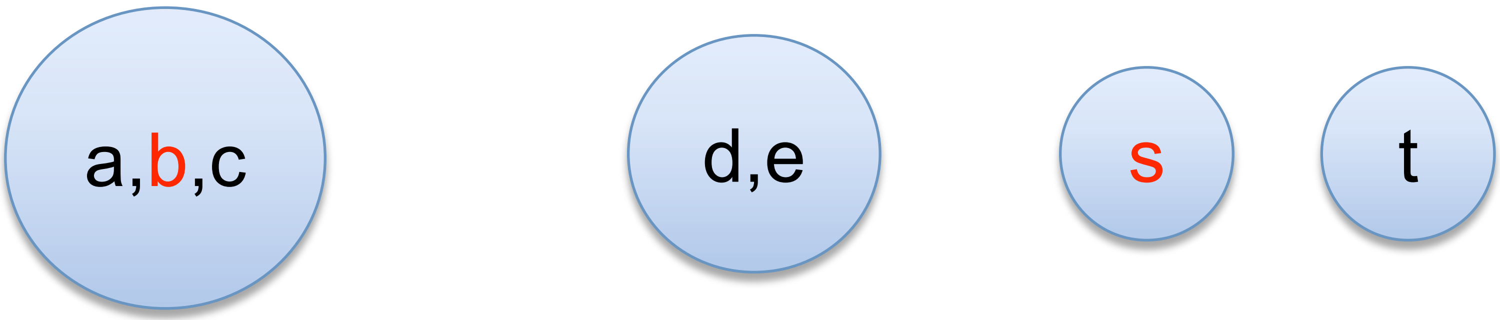
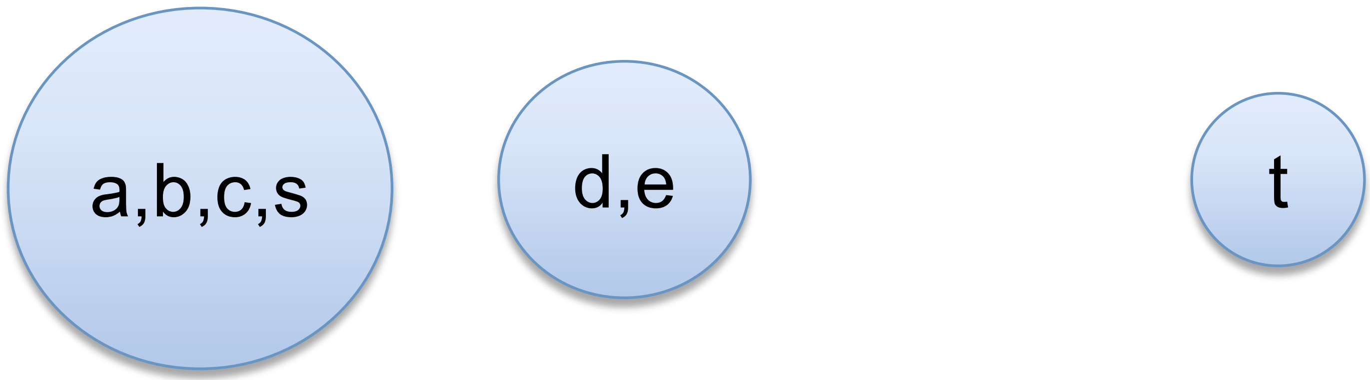


Deciding Equality

$$a = b,\ b = c,\ d = e,\ b = s,\ d = t,\ a \neq e,\ a \neq s$$



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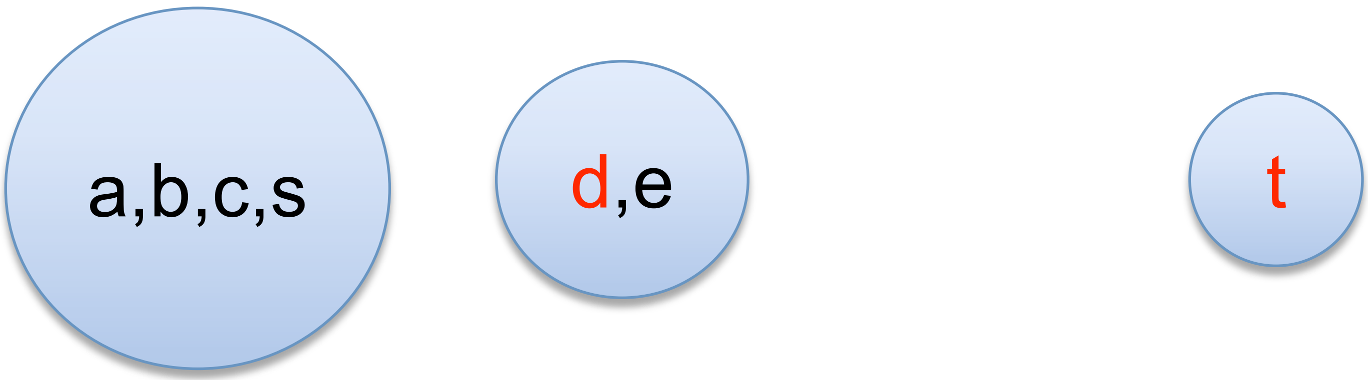

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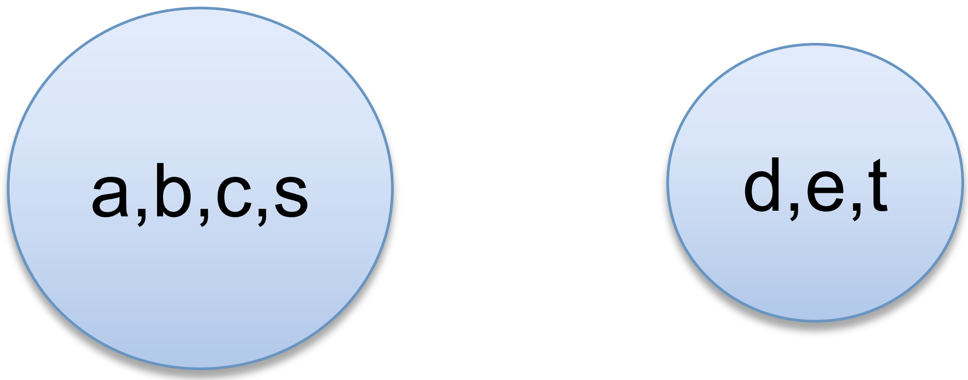

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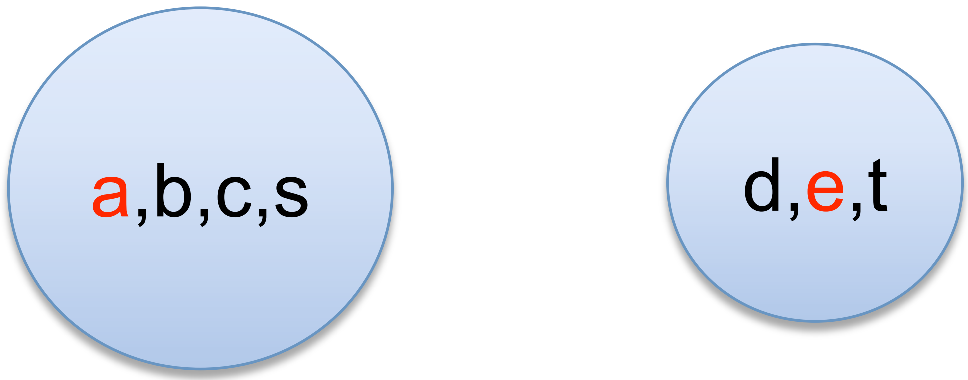

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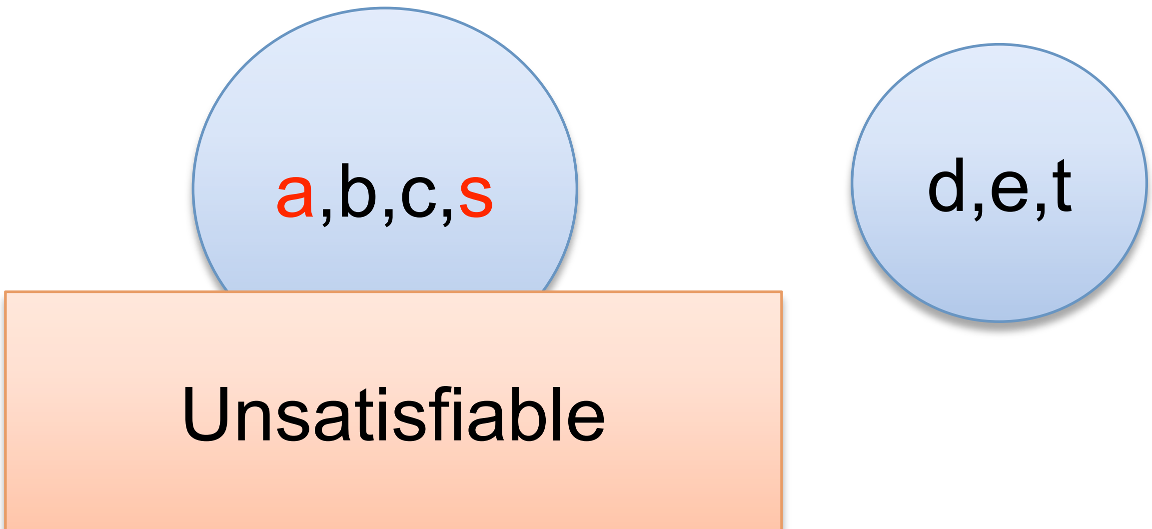
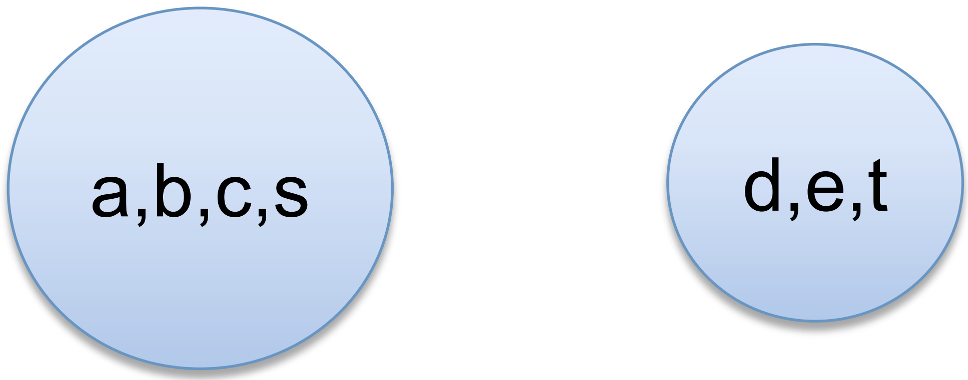


Deciding Equality

$a = b,\ b = c,\ d = e,\ b = s,\ d = t,\ a \neq e$



Model

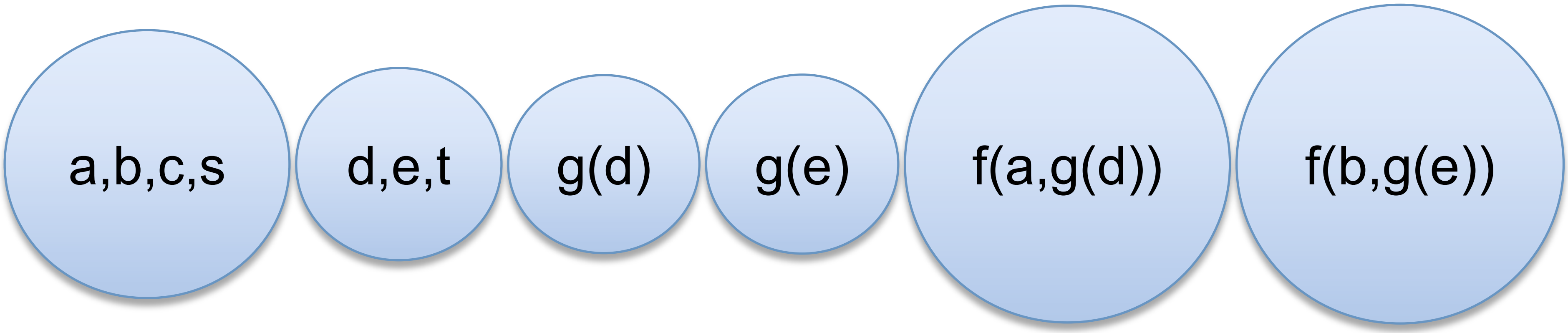
$|M| = \{\, 0, 1 \,\}$

$M(a) = M(b) = M(c) = M(s) = 0$

$M(d) = M(e) = M(t) = 1$

Deciding Equality + (uninterpreted) Functions

$a = b,\ b = c,\ d = e,\ b = s,\ d = t,\ f(a, g(d)) \neq f(b, g(e))$



Congruence Rule:

$x_1 = y_1, \ldots, x_n = y_n$ implies $f(x_1, \ldots, x_n) = f(y_1, \ldots, y_n)$

Deciding Equality + (uninterpreted) Functions

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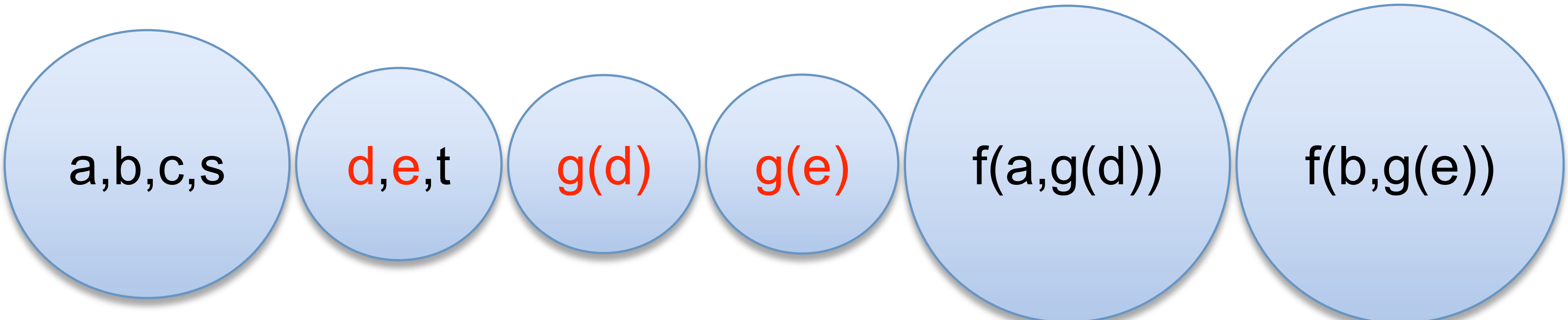

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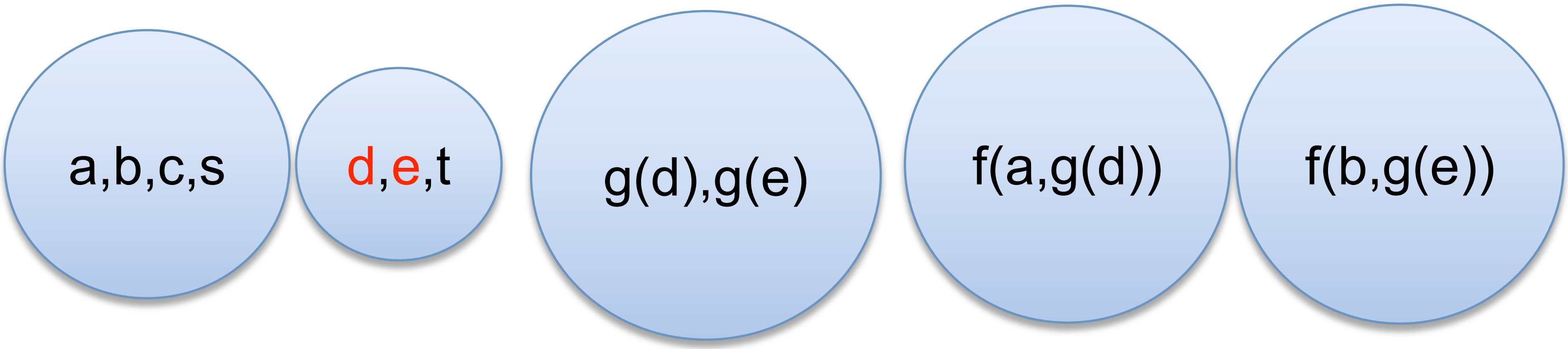

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Deciding Equality + (uninterpreted) Functions

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Deciding Equality + (uninterpreted) Functions

$$a = b,\ b = c,\ d = e,\ b = s,\ d = t,\ f(a, g(d)) \neq f(b, g(e))$$

Congruence Rule:

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Deciding Equality + (uninterpreted) Functions

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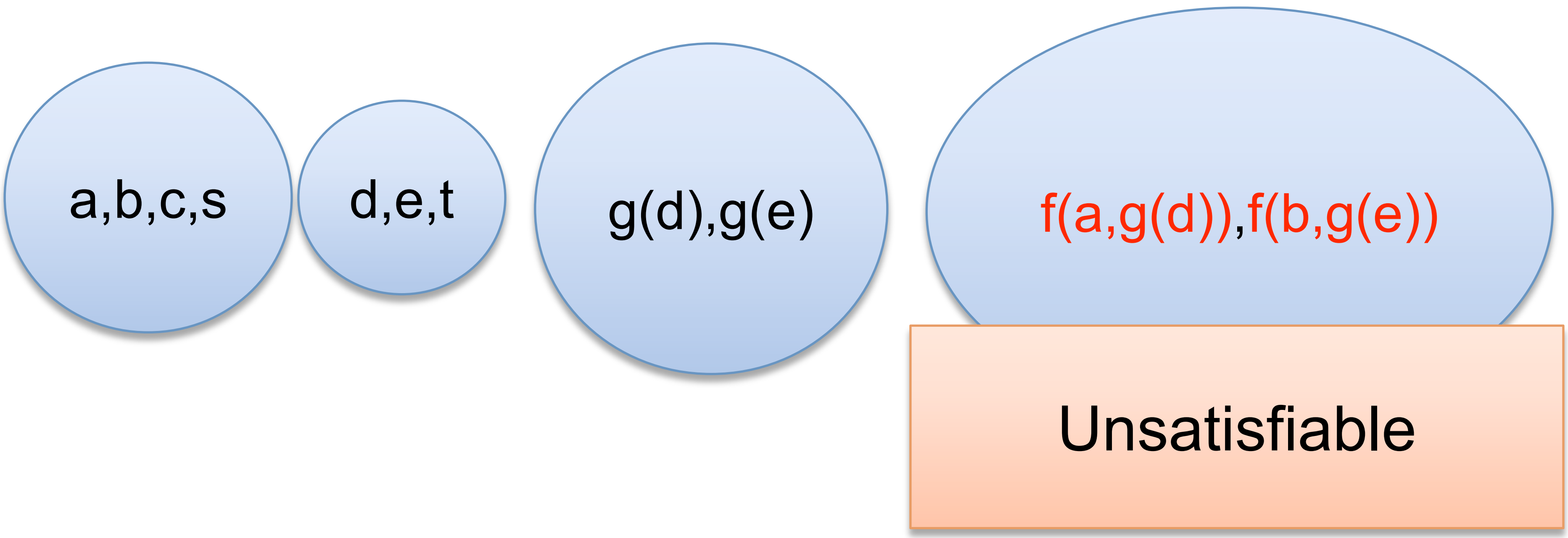

Deciding Equality + (uninterpreted) Functions

(fully shared) DAGs for representing terms

Union-find data-structure + Congruence Closure

$O(n \log n)$

Combining Solvers

In practice, we need a combination of theory solvers.

Nelson-Oppen combination method.

Reduction techniques.

Model-based theory combination.

SAT (propositional checkers): Case Analysis

$$p \vee q,$$
$$p \vee \neg q,$$
$$\neg p \vee q,$$
$$\neg p \vee \neg q$$

SAT (propositional checkers): Case Analysis

$p \vee q$,
$p \vee \neg q$,
$\neg p \vee q$,
$\neg p \vee \neg q$

Assignment:
p = false,
q = false

SAT (propositional checkers): Case Analysis

$p \vee q$,
$p \vee \neg q$,
$\neg p \vee q$,
$\neg p \vee \neg q$

Assignment:
p = false,
q = true

SAT (propositional checkers): Case Analysis

$$
\begin{aligned}
p \lor q&, \\
p \lor \neg q&, \\
\neg p \lor q&, \\
\neg p \lor \neg q&
\end{aligned}
$$

Assignment:
p = true,
q = false

SAT (propositional checkers): Case Analysis

$p \lor q$,
$p \lor \neg q$,
$\neg p \lor q$,
$\neg p \lor \neg q$

Assignment:
p = true,
q = true

DPLL

M | F

Partial model

Set of clauses

DPLL

- Guessing

$$p \;|\; p \vee q, \neg q \vee r$$

$$\Downarrow$$

$$p, \neg q \;|\; p \vee q, \neg q \vee r$$

DPLL

- Deducing

$$p \;\; | \;\; p \vee q, \neg p \vee s$$

$$\Downarrow$$

$$p, s \, | \; p \vee q, \neg p \vee s$$

DPLL

- Backtracking

$$p, \neg s, q \mid p \vee q, s \vee q, \neg p \vee \neg q$$

$$p, s \mid p \vee q, s \vee q, \neg p \vee \neg q$$

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Modern DPLL

- Efficient indexing (two-watch literal)
- Non-chronological backtracking (backjumping)
- Lemma learning
- …

Solvers = DPLL + Decision Procedures

- Efficient decision procedures for conjunctions of ground literals.

$$a=b, a<5 \mid \neg a=b \lor f(a)=f(b), \quad a < 5 \lor a > 10$$

Theory Conflicts

a=b, a > 0, c > 0, a + c < 0 | F

backtrack

Naïve recipe?

SMT Solver = DPLL + Decision Procedure

Standard question:

Why don't you use CPLEX for handling linear arithmetic?

Efficient SMT solvers

Decision Procedures must be:

Incremental & Backtracking

Theory Propagation

$$a=b,\ a<5 \mid \ldots\ a<6 \vee f(a) = a$$

$$\Downarrow$$

$$a=b,\ a<5,\ a<6 \mid \ldots\ a<6 \vee f(a) = a$$

Efficient SMT solvers

Decision Procedures must be:

- Incremental & Backtracking
- Theory Propagation
- Precise (theory) lemma learning
  - $a=b,\ a > 0,\ c > 0,\ a + c < 0 \mid F$
  - Learn clause:
  - $\neg(a=b) \lor \neg(a > 0) \lor \neg(c > 0) \lor \neg(a + c < 0)$
  - Imprecise!
  - Precise clause:
  - $\neg a > 0 \lor \neg c > 0 \lor \neg a + c < 0$

SMT x SAT

For some theories, SMT can be reduced to SAT

Higher level of abstraction

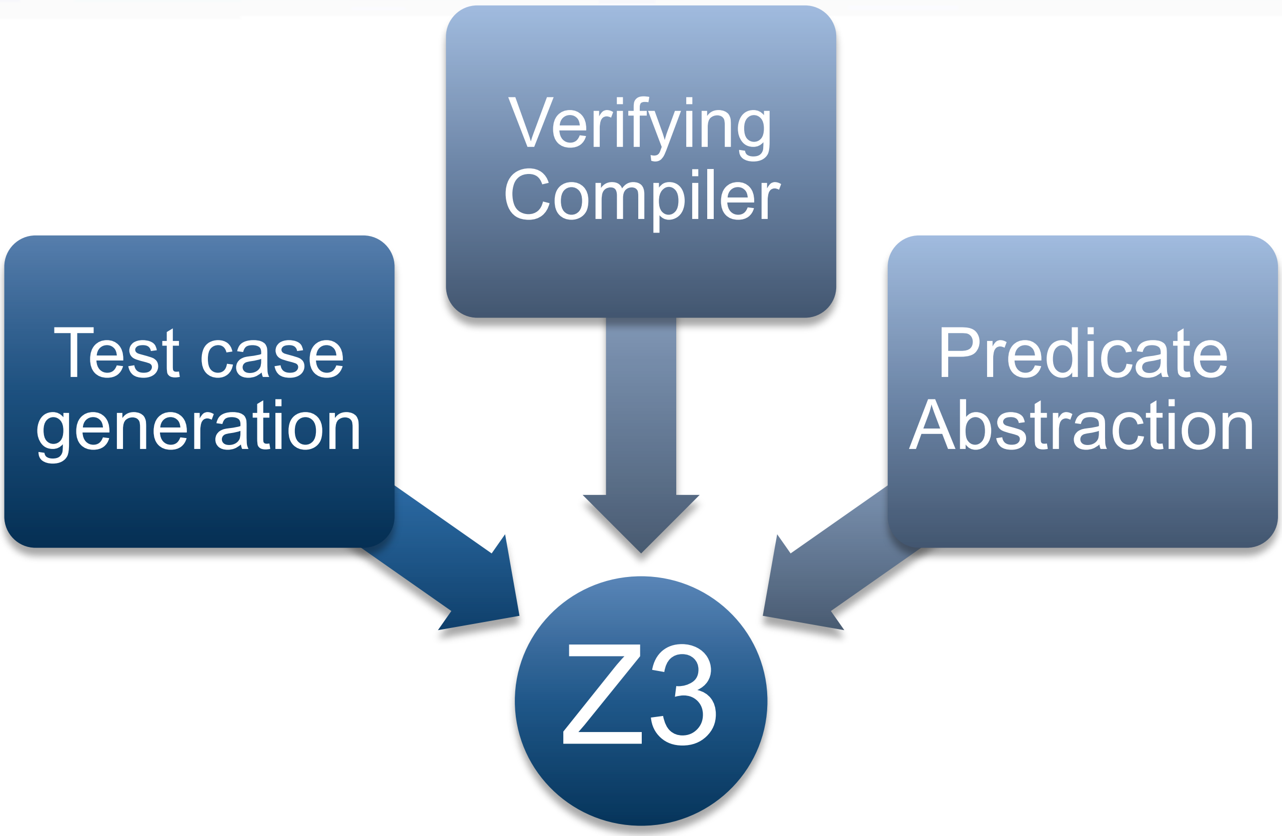
$$\mathbf{bvmul_{32}(a,b) = bvmul_{32}(b,a)}$$

SMT x First-order provers

$F \cup T$ → First-order Theorem Prover

T may not have a finite axiomatization

SMT: Some Applications



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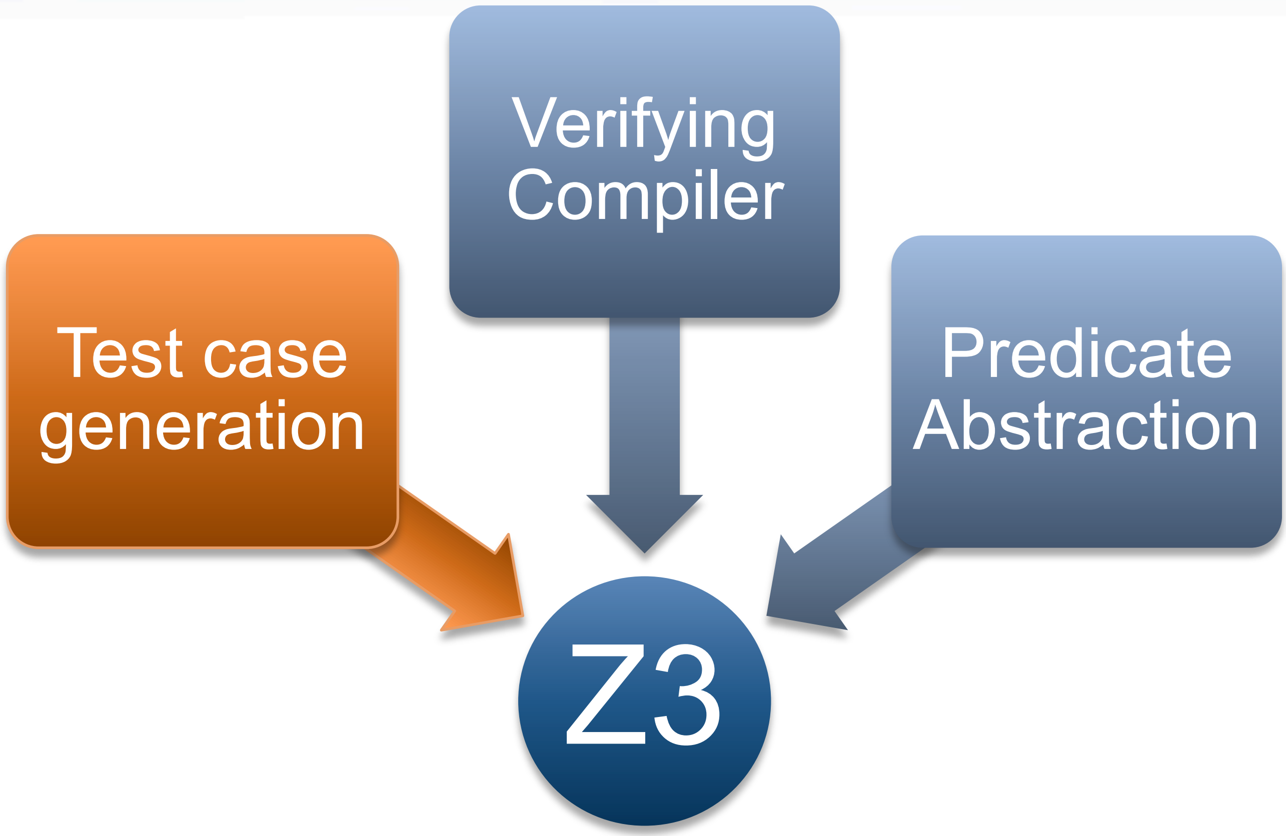
SMT: Some Applications



Test-case generation

- Test (correctness + usability) is 95% of the deal:
  - Dev/Test is 1-1 in products.
  - Developers are responsible for unit tests.
- Tools:
  - Annotations and static analysis (SAL + ESP)
  - File Fuzzing
  - Unit test case generation

Security is critical

- Security bugs can be very expensive:
  - Cost of each MS Security Bulletin: $600k to $Millions.
  - Cost due to worms: $Billions.
  - The real victim is the customer.
- Most security exploits are initiated via files or packets.
  - Ex: Internet Explorer parses dozens of file formats.
- Security testing: hunting for million dollar bugs
  - Write A/V
  - Read A/V
  - Null pointer dereference
  - Division by zero

Hunting for Security Bugs.

- Two main techniques used by *"black hats"*:
  - Code inspection (of binaries).
  - *Black box fuzz testing.*

- **Black box** fuzz testing:
  - A form of black box random testing.
  - Randomly *fuzz* (=modify) a well formed input.
  - Grammar-based fuzzing: rules to encode how to fuzz.
- **Heavily** used in security testing
  - At MS: several internal tools.
  - Conceptually simple yet effective in practice

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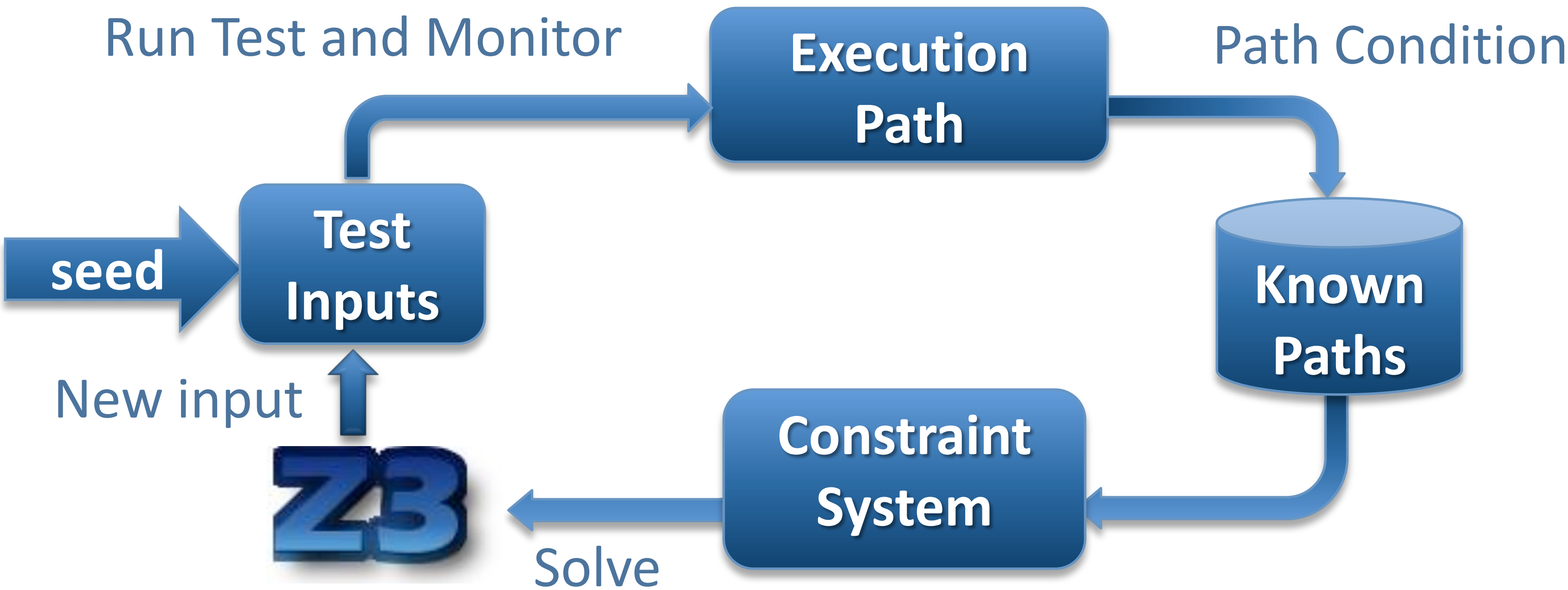
Directed Automated Random Testing (DART)



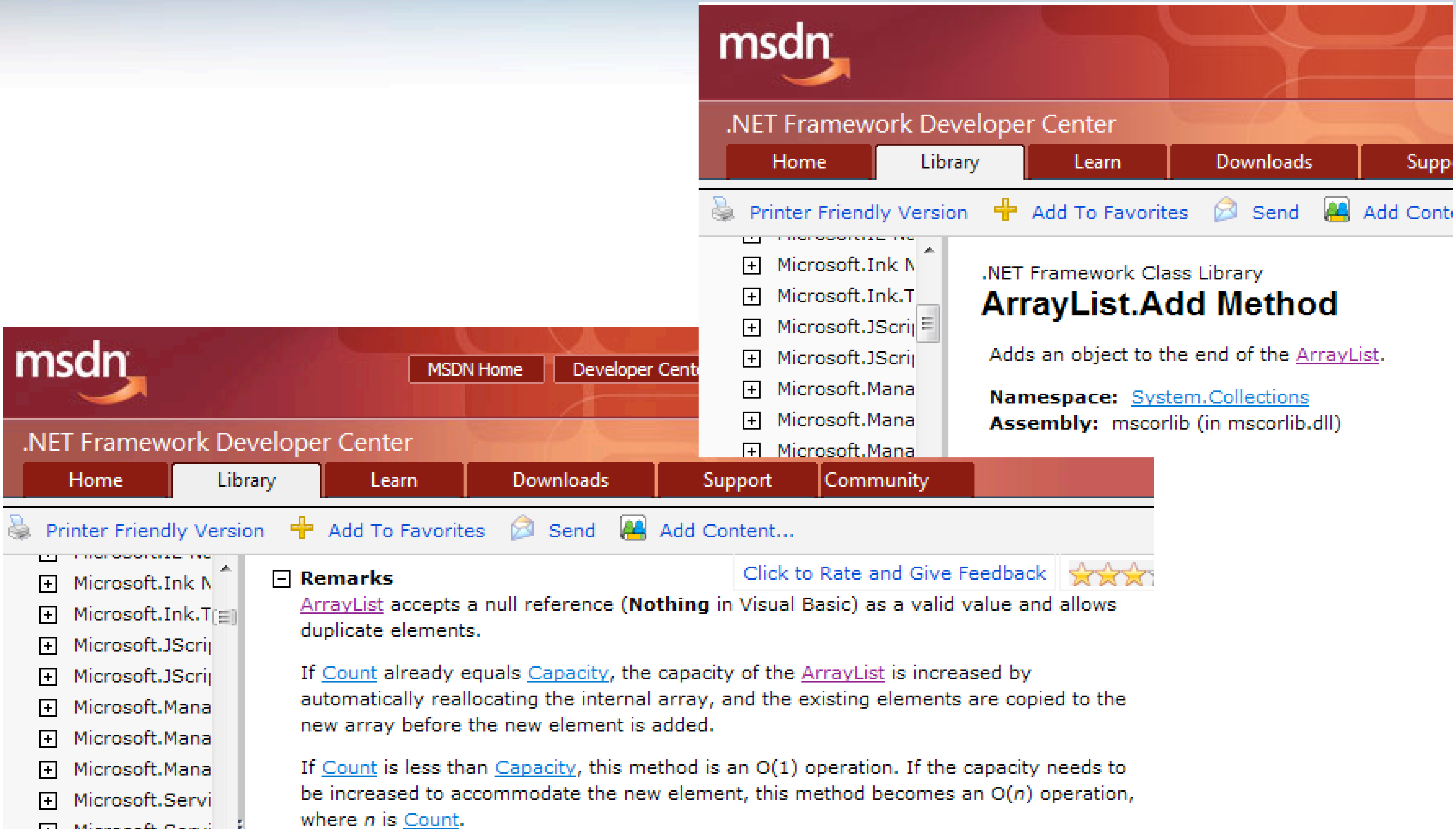
DARTish projects at Microsoft

PEX	Implements DART for .NET.
SAGE	Implements DART for x86 binaries.
YOGI	Implements DART to check the feasibility of program paths generated statically using a SLAM-like tool.
Vigilante	Partially implements DART to dynamically generate worm filters.

What is Pex?

- Test input generator
  - Pex starts from parameterized unit tests
  - Generated tests are emitted as traditional unit tests

ArrayList: The Spec



ArrayList: AddItem Test

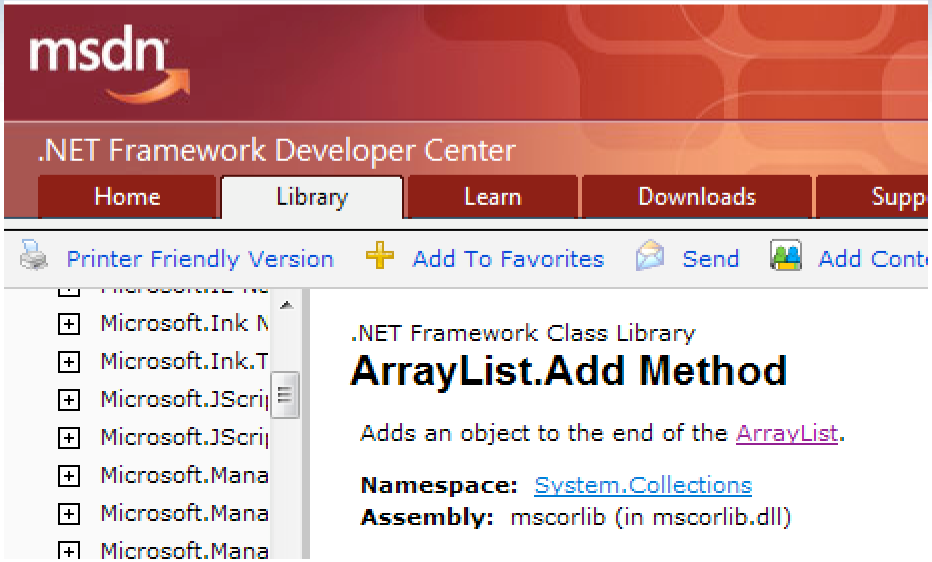
```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```



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ArrayList: Starting Pex…

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
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...
```

Inputs

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ArrayList: Run 1, (0,null)

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
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Inputs
(0,null)

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      var list = new ArrayList(c);
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      Assert(list[0] == item); }
}
```

	Inputs	Observed Constraints
	(0,null)	!(c<0)

```
class ArrayList {
  object[] items;
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  ArrayList(int capacity) {
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    items = new object[capacity];
  }

  void Add(object item) {
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      ResizeArray();

    items[this.count++] = item; }
...
```

c < 0 → false

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ArrayList: Run 1, (0,null)

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
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      ResizeArray();

    items[this.count++] = item; }
...
```

0 == c → true

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ArrayList: Run 1, (0,null)

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

```
item == item → true
```

	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
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  }

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...
```

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ArrayList: Picking the next branch to cover

```
class ArrayListTest {
  [PexMethod]
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      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

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  void Add(object item) {
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      ResizeArray();

    items[this.count++] = item; }
...
```

Constraints to solve	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c
!(c<0) && **0!=c**		

ArrayList: Solve constraints using SMT solver

```
class ArrayListTest {
  [PexMethod]
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      var list = new ArrayList(c);
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      Assert(list[0] == item); }
}
```

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  void Add(object item) {
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    items[this.count++] = item; }
...
```

Constraints to solve	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c
!(c<0) && 0!=c	**(1,null)**	

Microsoft Research

ArrayList: Run 2, (1, null)

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

Constraints to solve	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c
!(c<0) && 0!=c	(1,null)	!(c<0) && 0!=c

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```

0 == c → false

Microsoft Research

ArrayList: Pick new branch

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```

Constraints to solve	Inputs	Observed Constraints
	`(0,null)`	`!(c<0) && 0==c`
`!(c<0) && 0!=c`	`(1,null)`	`!(c<0) && 0!=c`
`c<0`		

Z3

Microsoft Research

ArrayList: Run 3, (-1, null)

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```

Constraints to solve	Inputs	Observed Constraints
	`(0,null)`	`!(c<0) && 0==c`
`!(c<0) && 0!=c`	`(1,null)`	`!(c<0) && 0!=c`
`c<0`	**`(-1,null)`**	

ArrayList: Run 3, (-1, null)

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

Constraints to solve	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c
!(c<0) && 0!=c	(1,null)	!(c<0) && 0!=c
c<0	**(-1,null)**	c<0

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```

c < 0 → true

Microsoft Research

ArrayList: Run 3, (-1, null)

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```

Constraints to solve	Inputs	Observed Constraints
	`(0,null)`	`!(c<0) && 0==c`
`!(c<0) && 0!=c`	`(1,null)`	`!(c<0) && 0!=c`
`c<0`	`(-1,null)`	`c<0`

PEX ↔ Z3

Rich Combination → Linear arithmetic → Bitvector → Arrays → Free Functions

Models → Model used as test inputs

∀-Quantifier → Used to model custom theories (e.g., .NET type system)

API → Huge number of small problems. Textual interface is too inefficient.

PEX ⟷ Z3: Incrementality

- Pex “sends” several similar formulas to Z3.
- Plus: backtracking primitives in the Z3 API.
  - **push**
  - **pop**
- Reuse (some) lemmas.

PEX ↔ Z3: Small models

- **Given** a set of constraints *C*, find a model *M* that minimizes the interpretation for $x_0, \ldots, x_n$.
- In the ArrayList example:
  - Why is the model where *c* = 2147483648 less desirable than the model with *c* = 1?

```
!(c<0) && 0!=c
```

- Simple solution:

  Assert *C*

  while satisfiable

  Peek x_i such that $M[x_i]$ is big

  Assert $x_i < n$, where n is a small constant

  Return last found model

PEX ⟷ Z3: Small models

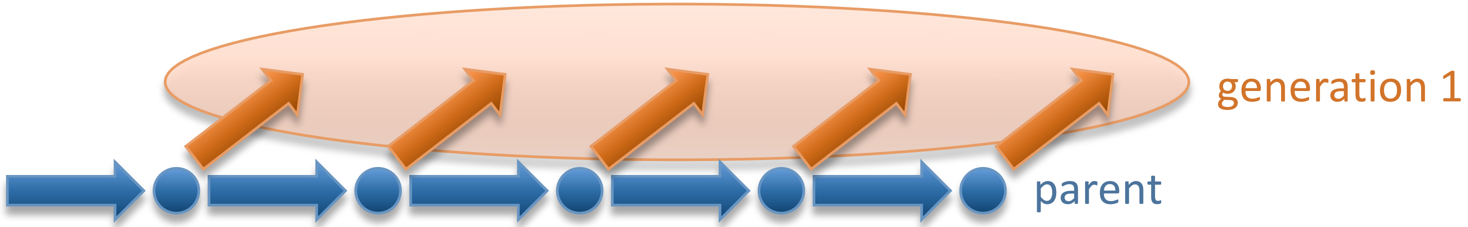
- **Given** a set of constraints *C*, find a model *M* that minimizes the interpretation for $x_0, \ldots, x_n$.
- In the ArrayList example:
  - Why is the model where *c* = 2147483648 less desirable than the model with *c* = 1?

```
!(c<0) && 0!=c
```

- **Refinement**:
  - Eager solution stops as soon as the system becomes unsatisfiable.
  - A "bad" choice (peek x_i) may prevent us from finding a good solution.
  - Use **push** and **pop** to retract "bad" choices.

SAGE

- Apply DART to large applications (not units).
- Start with well-formed input (not random).
- Combine with generational search (not DFS).
  - Negate 1-by-1 each constraint in a path constraint.
  - Generate many children for each parent run.



Microsoft Research

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000040h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 0 – seed file

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 00 00 00 00 00 00 00 00 00 00 00 00 ; RIFF............
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000040h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 1

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 00 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF....***....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000040h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 2

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 3D 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF=...*** ....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000040h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 3

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 3D 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF=...*** ....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 73 74 72 68 00 00 00 00 00 00 00 00 ; ....strh........
00000040h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 4

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 3D 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF=...*** ....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 73 74 72 68 00 00 00 00 76 69 64 73 ; ....strh....vids
00000040h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 5

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 3D 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF=...*** ....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 73 74 72 68 00 00 00 00 76 69 64 73 ; ....strh....vids
00000040h: 00 00 00 00 73 74 72 66 00 00 00 00 00 00 00 00 ; ....strf........
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 6

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 3D 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF=...*** ....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 73 74 72 68 00 00 00 00 76 69 64 73 ; ....strh....vids
00000040h: 00 00 00 00 73 74 72 66 00 00 00 00 28 00 00 00 ; ....strf....(...
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 7

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 3D 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF=...*** ....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 73 74 72 68 00 00 00 00 76 69 64 73 ; ....strh....vids
00000040h: 00 00 00 00 73 74 72 66 00 00 00 00 28 00 00 00 ; ....strf....(...
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 C9 9D E4 4E ; ............ÉäN
00000060h: 00 00 00 00                                     ; ....
```

Generation 8

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 3D 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF=...*** ....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 73 74 72 68 00 00 00 00 76 69 64 73 ; ....strh....vids
00000040h: 00 00 00 00 73 74 72 66 00 00 00 00 28 00 00 00 ; ....strf....(...
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 01 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 9

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 3D 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF=...*** ....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 73 74 72 68 00 00 00 00 76 69 64 73 ; ....strh....vids
00000040h: 00 00 00 00 73 74 72 66 B2 75 76 3A 28 00 00 00 ; ....strf²uv:(...
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 01 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 10 – CRASH

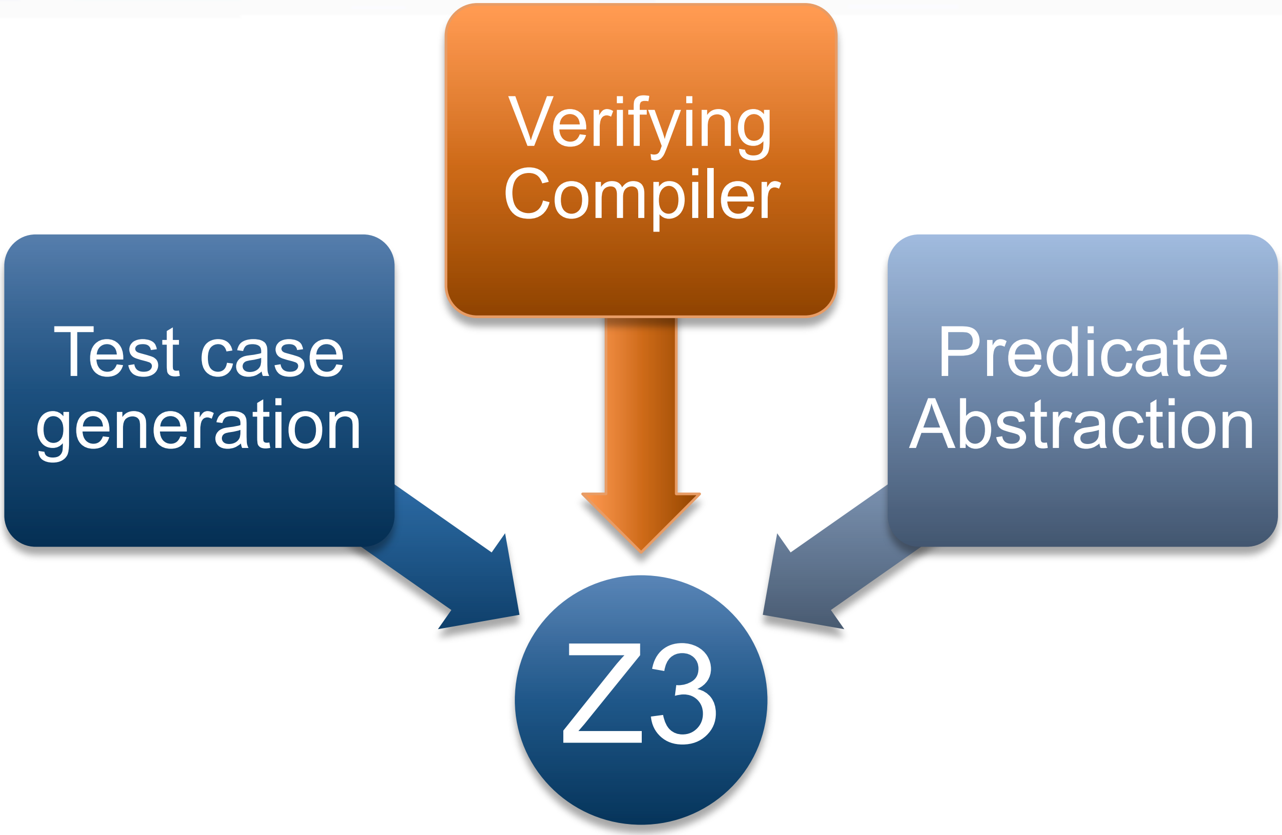
SAGE (cont.)

- SAGE is very effective at finding bugs.
- Works on large applications.
- Fully automated
- Easy to deploy (x86 analysis – any language)
- Used in various groups inside Microsoft
- Powered by Z3.

SAGE ↔ Z3

- Formulas are usually big conjunctions.
- SAGE uses only the bitvector and array theories.
- Pre-processing step has a huge performance impact.
  - Eliminate variables.
  - Simplify formulas.
- Early unsat detection.

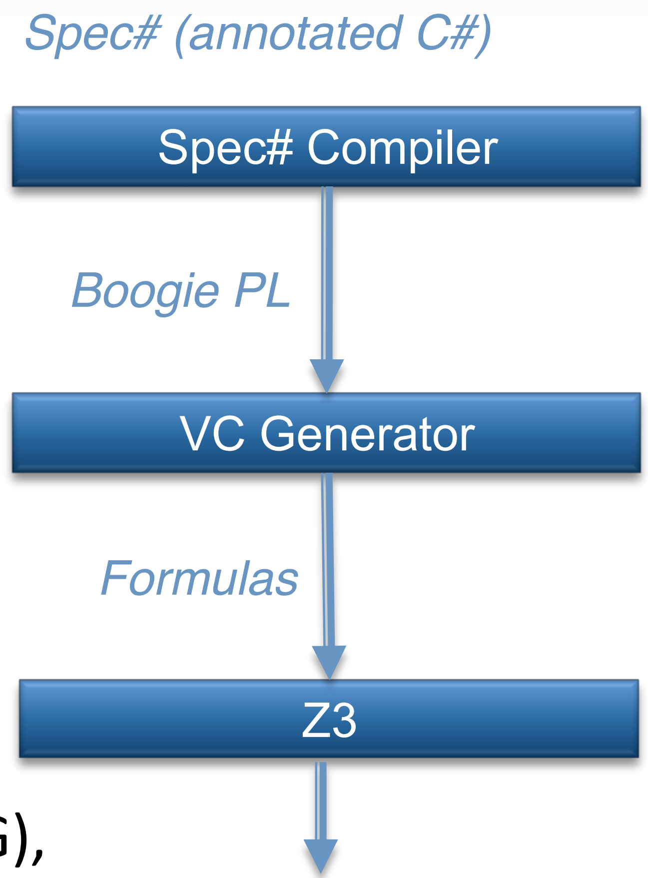
SMT: Some Applications



Microsoft Research

Spec# Approach for a Verifying Compiler

- *Source Language*
  - C# + goodies = Spec#
- *Specifications*
  - method contracts,
  - invariants,
  - field and type annotations.
- *Program Logic:*
  - *Dijkstra's weakest preconditions.*
- *Automatic Verification*
  - type checking,
  - verification condition generation (VCG),
  - automatic theorem proving Z3



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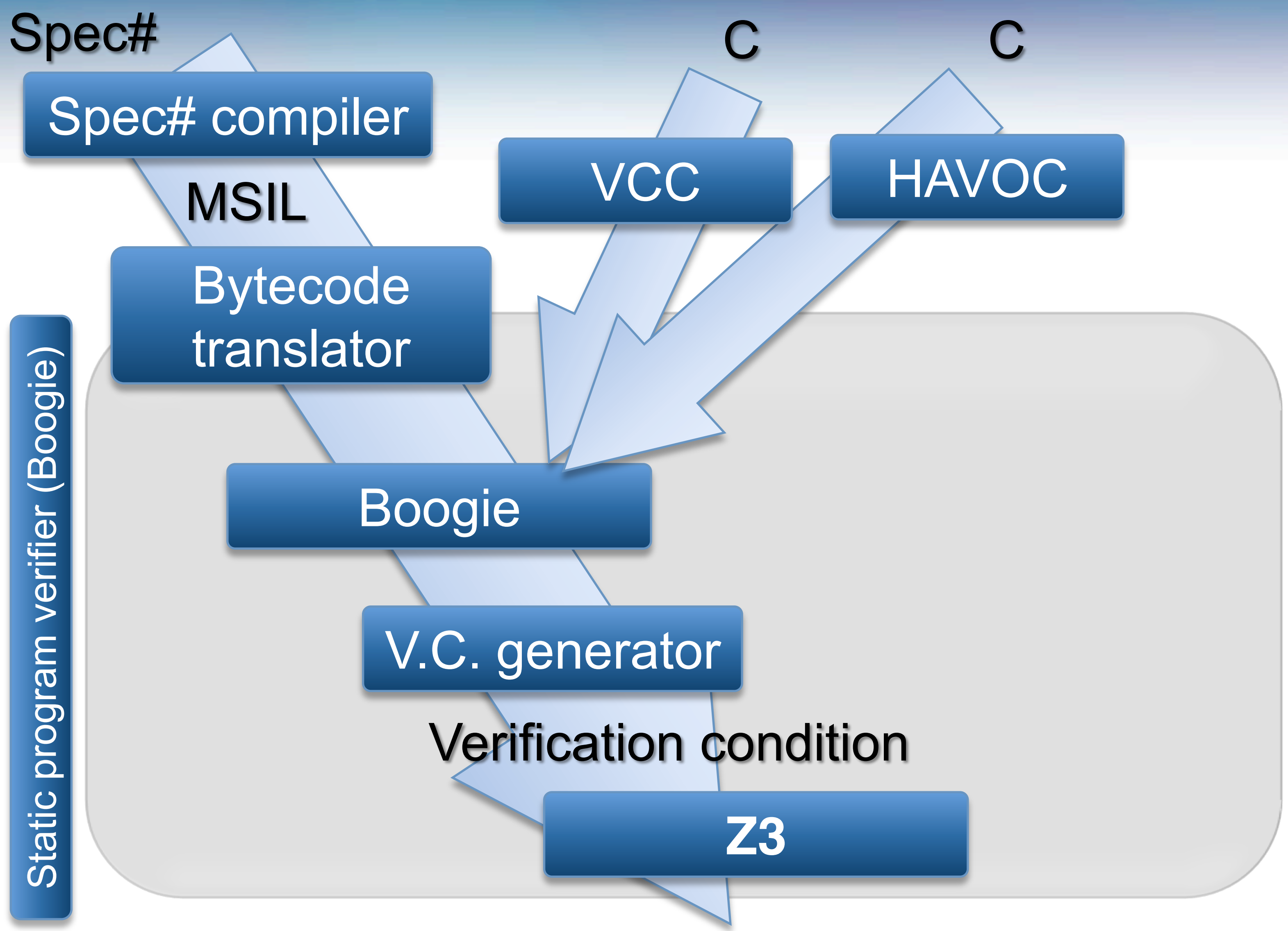
Verification architecture



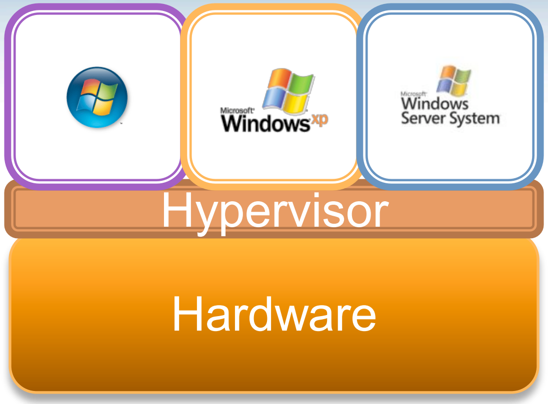
HAVOC

- A tool for specifying and checking properties of systems software written in C.
- It also translates annotated C into Boogie PL.
- It allows the expression of *richer properties about the program heap and data structures* such as linked lists and arrays.
- HAVOC is being used to specify and check:
  - Complex locking protocols over heap-allocated data structures in Windows.
  - Properties of collections such as IRP queues in device drivers.
  - Correctness properties of custom storage allocators.

A Verifying C Compiler

- VCC translates an *annotated C program* into a *Boogie PL* program.
- A C-ish memory model
  - Abstract heaps
  - Bit-level precision
- Microsoft Hypervisor: verification grand challenge.

Hypervisor: A Manhattan Project



- **Meta OS**: small layer of software between hardware and OS
- **Mini**: 60K lines of non-trivial concurrent systems C code
- **Critical:** must provide functional resource abstraction
- **Trusted**: a verification grand challenge

Hypervisor: Some Statistics

- VCs have several Mb
- Thousands of non ground clauses
- Developers are willing to wait at most 5 min per VC

Main Challenge

- Quantifiers, quantifiers, quantifiers, …
- Modeling the runtime

$$\forall\ h,o,f:$$
$$IsHeap(h) \wedge o \neq null \wedge read(h, o, alloc) = t$$
$$\Rightarrow$$
$$read(h,o, f) = null \vee read(h, read(h,o,f),alloc) = t$$

Main Challenge

- Quantifiers, quantifiers, quantifiers, …
- Modeling the runtime
- Frame axioms

$$\forall\ o, f:$$
$$o \neq null \wedge read(h_0, o, alloc) = t \Rightarrow$$
$$read(h_1,o,f) = read(h_0,o,f) \vee (o,f) \in M$$

Main Challenge

- Quantifiers, quantifiers, quantifiers, …
- Modeling the runtime
- Frame axioms
- User provided assertions

  $\forall\ i,j\!: i \leq j \Rightarrow read(a,i) \leq read(b,j)$

Main Challenge

- Quantifiers, quantifiers, quantifiers, …
- Modeling the runtime
- Frame axioms
- User provided assertions
- Theories

$\forall$ x: p(x,x)

$\forall$ x,y,z: p(x,y), p(y,z) $\Rightarrow$ p(x,z)

$\forall$ x,y: p(x,y), p(y,x) $\Rightarrow$ x = y

Main Challenge

- Quantifiers, quantifiers, quantifiers, …
- Modeling the runtime
- Frame axioms
- User provided assertions
- Theories
- Solver must be fast in satisfiable instances.

We want to find bugs!

Bad news

There is no sound and refutationally complete procedure for linear integer arithmetic + free function symbols

Many Approaches

- Heuristic quantifier instantiation
- Combining SMT with Saturation provers
- Complete quantifier instantiation
- Decidable fragments
- Model based quantifier instantiation

Challenge: modeling runtime

- Is the axiomatization of the runtime consistent?
- False implies everything
- Partial solution: SMT + Saturation Provers
- Found many bugs using this approach

Challenge: Robustness

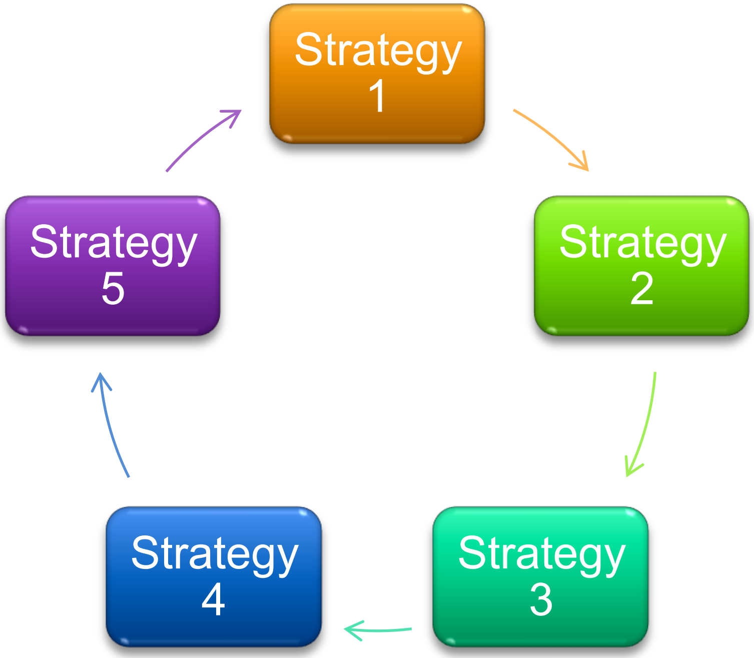
- Standard complain

  "I made a small modification in my Spec, and Z3 is timingout"

- This also happens with SAT solvers (NP-complete)
- In our case, the problems are undecidable
- Partial solution: parallelization

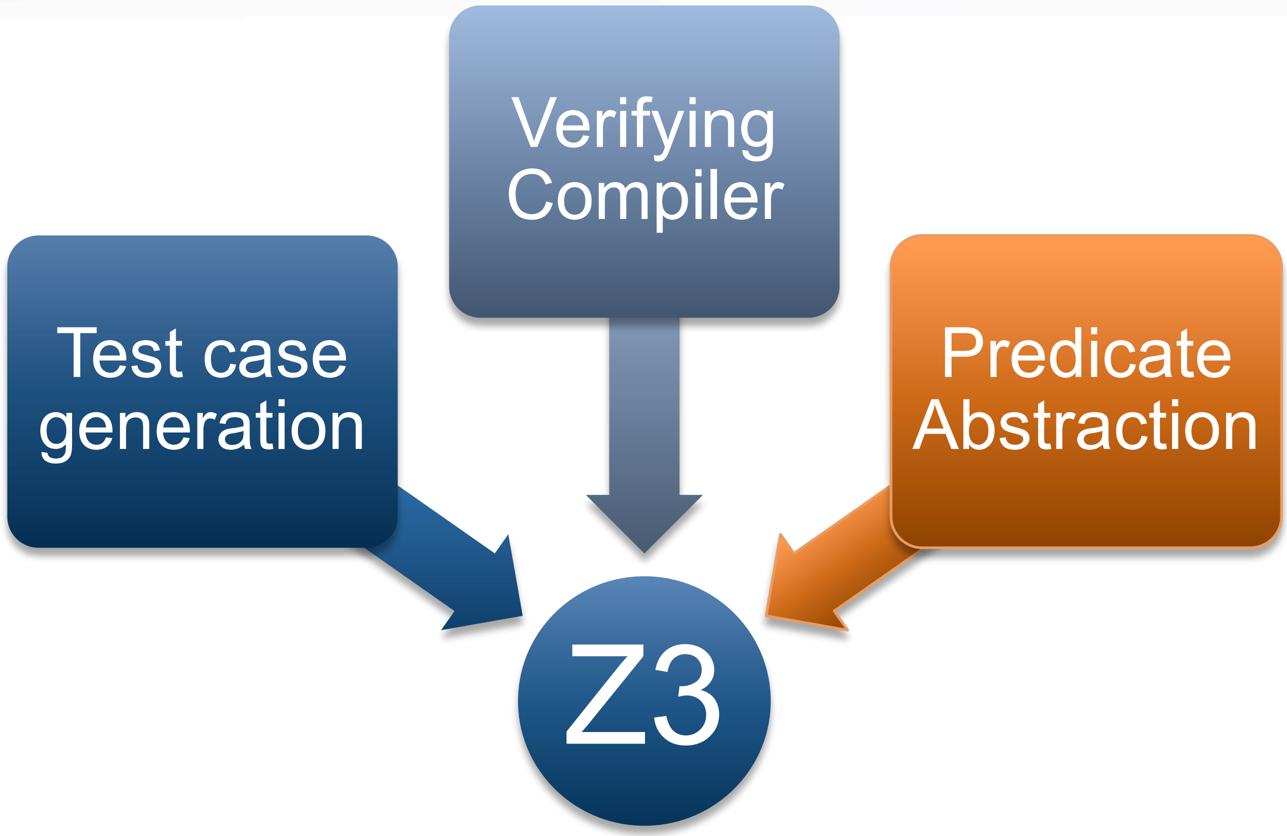
Parallel Z3

- Joint work with Y. Hamadi (MSRC) and C. Wintersteiger
- Multi-core & Multi-node (HPC)
- Different strategies in parallel
- Collaborate exchanging lemmas



Microsoft Research

SMT: Some Applications



Overview

- *http://research.microsoft.com/slam/*
- *SLAM/SDV* is a software model checker.
- Application domain: *device drivers*.
- Architecture:

  c2bp C program → boolean program (*predicate abstraction).*

  bebop Model checker for boolean programs.

  newton Model refinement (check for path feasibility)
- SMT solvers are used to perform predicate abstraction and to check path feasibility.
- c2bp makes several calls to the SMT solver. The formulas are relatively small.

Predicate Abstraction: *c2bp*

- **Given** a C program P and $F = \{p_1, \ldots, p_n\}$.
- **Produce** a Boolean program $B(P, F)$
  - Same control flow structure as P.
  - Boolean variables $\{b_1, \ldots, b_n\}$ to match $\{p_1, \ldots, p_n\}$.
  - Properties true in $B(P, F)$ are true in P.
- Each p_i is a pure Boolean expression.
- Each p_i represents set of states for which p_i is true.
- Performs modular abstraction.

Abstracting Expressions via *F*

- $Implies_F\,(e)$
  - Best Boolean function over *F* that implies *e.*
- $ImpliedBy_F\,(e)$
  - Best Boolean function over *F* that is implied by *e.*
  - $ImpliedBy_F\,(e) = not\ Implies_F\,(not\ e)$

Computing $Implies_F(e)$

- minterm $m = l_1 \wedge \ldots \wedge l_n$, where $l_i = p_i$, or $l_i = not\ p_i$.
- $Implies_F\ (e)$: disjunction of all minterms that imply e.
- Naive approach
  - Generate all 2^n possible minterms.
  - For each minterm m, use SMT solver to check validity of $m \Rightarrow e$.
- Many possible optimizations

Computing $Implies_F(e)$

- $F = \{ x < y, x = 2\}$
- $e : y > 1$
- Minterms over F
  - $!x<y, !x=2$ implies $y>1$
  - $x<y, !x=2$ implies $y>1$
  - $!x<y, x=2$ implies $y>1$
  - $x<y, x=2$ implies $y>1$

$Implies_F(y>1) = x<y \wedge x=2$

Computing $Implies_F(e)$

- $F = \{ x < y, x = 2\}$
- $e : y > 1$
- Minterms over F
  - !x<y, !x=2 implies y>1
  - x<y, !x=2 implies y>1
  - !x<y, x=2 implies y>1
  - x<y, x=2 implies y>1

$Implies_F(y>1) = b_1 \wedge b_2$

Newton

- Given an error path *p* in the Boolean program *B*.
- Is *p* a feasible path of the corresponding C program?
  - Yes: found a bug.
  - No: find predicates that explain the infeasibility.
- Execute path symbolically.
- Check conditions for inconsistency using Z3.

SLAM ⟷ Z3

- All-SAT
  - Better (more precise) Predicate Abstraction
- Unsatisfiable cores
  - Why the abstract path is not feasible?
  - Fast Predicate Abstraction

SLAM ⟷ Z3: Unsatisfiable cores

- Let S be an unsatisfiable set of formulas.
- $S' \subseteq S$ is an unsatisfiable core of S if:
  - S' is also unsatisfiable, and
  - There is not $S'' \subset S'$ that is also unsatisfiable.
- Computing $\text{Implies}_F(e)$ with $F = \{p_1, p_2, p_3, p_4\}$
  - Assume $p_1, p_2, p_3, p_4 \Rightarrow e$ is valid
  - That is $p_1, p_2, p_3, p_4, \neg e$ is unsat
  - Now assume $p_1, p_3, \neg e$ is the unsatisfiable core
  - Then it is unnecessary to check:
    - $p_1, \neg p_2, p_3, p_4 \Rightarrow e$
    - $p_1, \neg p_2, p_3, \neg p_4 \Rightarrow e$
    - $p_1, p_2, p_3, \neg p_4 \Rightarrow e$

Other Microsoft clients

- Model programs (M. Veanes – MSRR)
- Termination (B. Cook – MSRC)
- Security protocols (A. Gordon and C. Fournet - MSRC)
- Business Application Modeling (E. Jackson - MSRR)
- Cryptography (R. Venki – MSRR)
- Verifying Garbage Collectors (C. Hawblitzel – MSRR)
- Model Based Testing (L. Bruck – SQL)
- Semantic type checking for D models (G. Bierman – MSRC)
- More coming soon…

Conclusion

- SMT is hot at Microsoft.
- Many applications.
- Z3 is a new and very efficient SMT solver.
- http://research.microsoft.com/projects/z3

Thank You!