

Streett Complementation Made Tight

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Outline

- 1 Introduction**
- 2 Preliminary
- 3 Lower Bound
- 4 Upper Bound
- 5 Conclusion

Automata-theoretical Model Checking

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Rabin Automata

Express fair termination, that is, all infinite computations be either unfair or eventually forever idling.

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Big Gap

For Streett and Rabin complementation, huge gaps exist as K can be as large as 2^N .

Previous 24 Hours

Lower Bound for Rabin Complementation

For any $\epsilon > 0$, there exists a constant $d > 0$ such that the lower bound of the complementation problem for Rabin automata over any alphabet of size d and with N states and K Rabin pairs is $2^{\Omega(KN \lg N)}$ if $K \leq 2^{N(1-\epsilon)}$, and is $2^{\Omega(2^{N(1-\epsilon)} N \lg N)}$ otherwise.

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Lower Bound for Rabin Determinization

These lower bounds also apply to Rabin determinization.

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Key Observation

The diversity of rankings on the run graphs of a T -automaton (automaton of type T) represents the complexity of the runs of the automaton.

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Since Klarlund 1991

Ranking has become a powerful weapon to attack the complementation problem, producing a sequence of sharper lower and upper bounds.

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Upper Bounds

A word is in the complementary language of a T automaton if and only if there exists an so-called odd co- T ranking on the run graph of the word. Complementation then amounts to constructing an automaton that can recognize the run graphs that admit the corresponding odd co- T rankings.

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- ☞ Cai, Zhang, Luo 2009: $2^{\Omega(KN \lg N)}$ for Rabin

Lower Bound Technique

Classic and Ad-Hoc

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👉 Intuitively appealing because it removes two layers of indirection.

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Trick II

Use an encoding method to bring down alphabet size

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Use **ranking functions** as a tool to establish certain properties of Δ -graphs.

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- Multi-dimensional rankings
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Essential Property

Co- T rankings transform the **universal** property that “all paths satisfy the co- T condition” to an **existential** property that “some path satisfies a T' condition”.

Our Contribution

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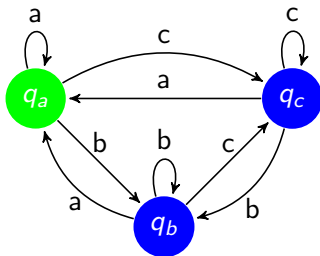
$T_{\mathcal{R}}$ and $T_{\mathcal{H}}$ are definite combinatorial numbers determined by K and N and bounded by $2^{O(N \lg N)}$.

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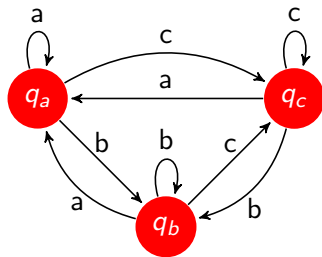
ω -Automata

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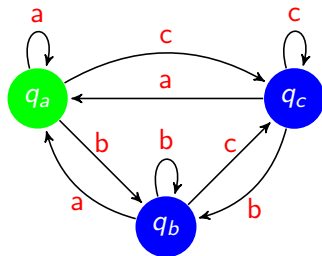
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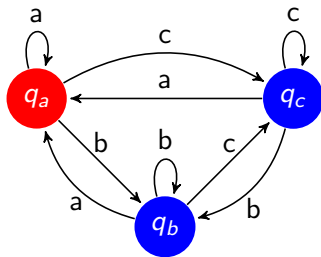
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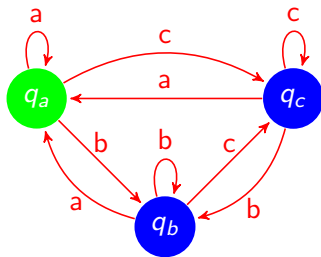
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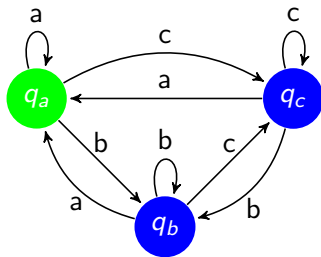
A (**nondeterministic**) ω -automaton \mathcal{A} is $\langle Q, \Sigma, I, \Delta, \mathcal{F} \rangle$:



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 - $\Delta \subseteq Q \times \Sigma \times Q$: a set of transitions;
- ☞ \mathcal{F} : the acceptance condition.

Δ -Graph

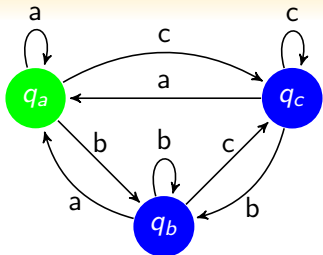
Let $\mathcal{A} = (\Sigma, Q, I, \Delta, \mathcal{F})$ be an automaton, and $\alpha = \sigma_1\sigma_2\cdots$ be an infinite word.

The Δ -graph of α under \mathcal{A} is a directed infinite graph $\mathcal{G} = (V, E)$ such that

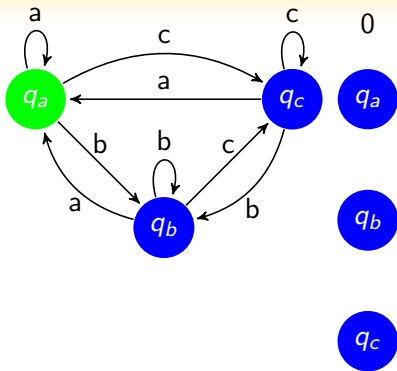
$$V = Q \times \mathbb{N}$$

$$E = \{ \langle \langle p, i \rangle, \langle q, i + 1 \rangle \rangle \mid p, q \in Q, i \in \mathbb{N} \text{ and } \langle p, \sigma_i, q \rangle \in \Delta \}.$$

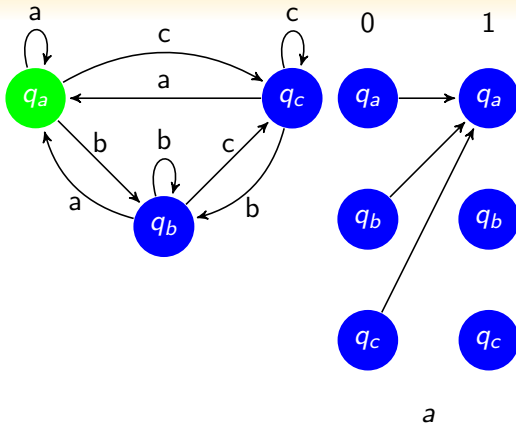
Example: Δ -Graph



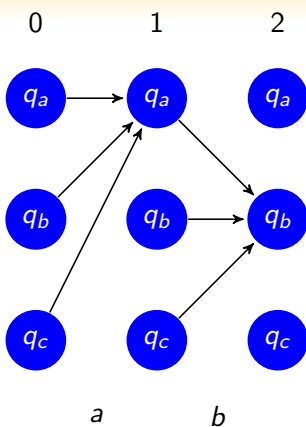
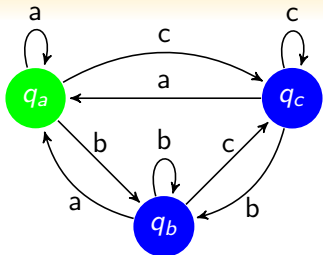
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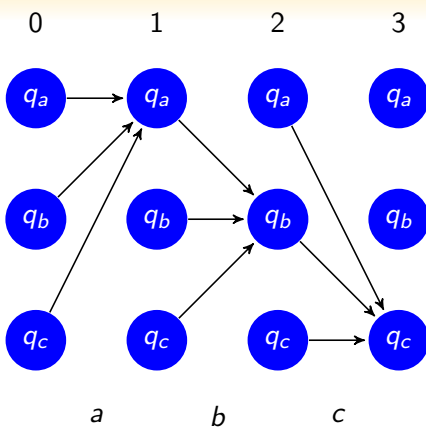
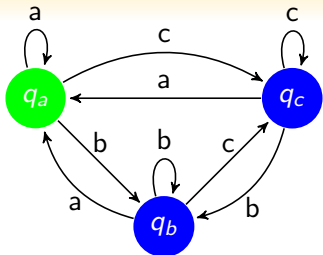
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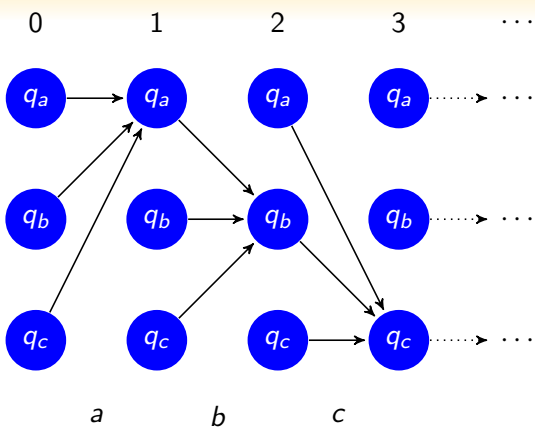
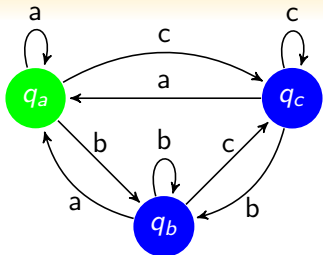
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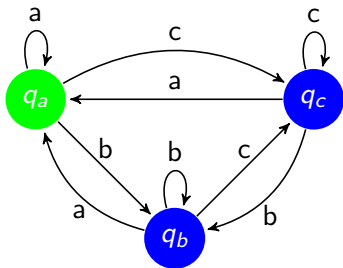
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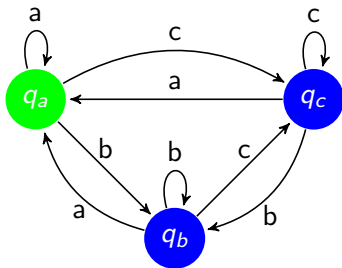
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Büchi Automata



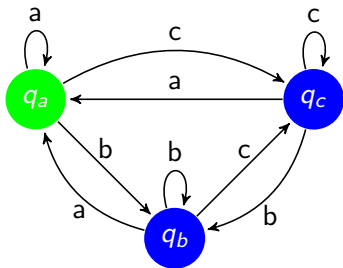
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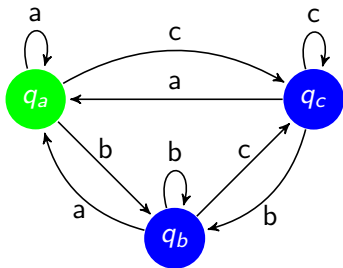


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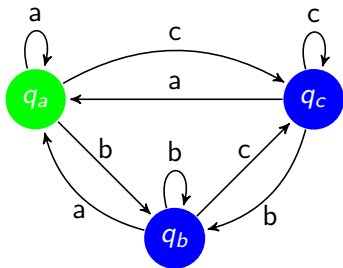
$$\mathcal{F} = \{q_b, q_c\}$$

$$\mathcal{L} = \{\alpha \in \{a, b, c\}^\omega \mid \exists^\omega b \in \alpha \text{ or } \exists^\omega c \in \alpha\}$$

Strejt Automata



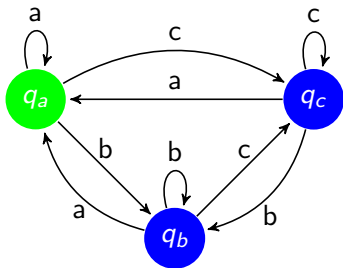
Streett Automata



Streett Condition

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Streett Automata

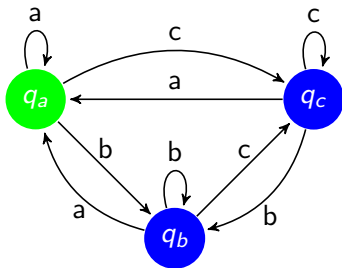


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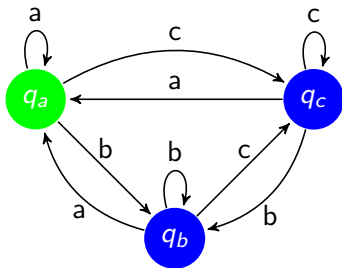
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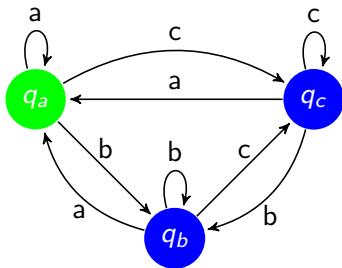
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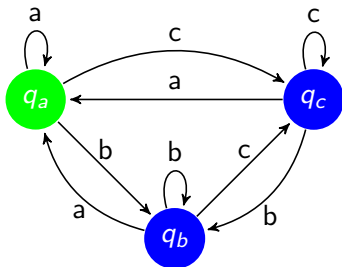
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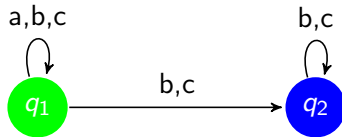
Complementation Problem

Input : an automaton \mathcal{A}

Output: an automaton \mathcal{B} such that $\mathcal{L}(\mathcal{B}) = \overline{\mathcal{L}(\mathcal{A})}$



$$\mathcal{F} = \{q_a\}$$

 \Rightarrow


$$\mathcal{F}' = \{q_2\}$$

Outline

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Proof Outline

- ☞ For each $k, n > 0$, we define a full Streett automaton $\mathcal{FS} = (\Sigma, S, Q, \Delta, \mathcal{F})$ and a set of Q -rankings $f : Q \rightarrow [1..n] \times I^k$.

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CONCLUSION

If a union-closed automaton \mathcal{CA} that complements \mathcal{FS} , then its state size is no less than the number of Q -rankings, because otherwise we can “weave” the runs of $(\mathcal{G}_f)^\omega$ and $(\mathcal{G}_{f'})^\omega$ in such a way that \mathcal{CA} would accept $((\mathcal{G}_f)^m(\mathcal{G}_{f'})^{m'})^\omega$ (for some $m, m' > 0$), contradicting $((\mathcal{G}_f)^+(\mathcal{G}_{f'})^+)^\omega \subseteq \mathcal{L}(\mathcal{FS})$.

Full Streett Automata

$\mathcal{FS} = (\Sigma, S, I, \Delta, \mathcal{F})$ such that

- 1 $S = Q \cup T \cup G \cup B$ where Q , T , G and B are pairwise disjoint and assume the following forms:

$$Q = \{q_0, \dots, q_{n-1}\},$$

$$T = \{t\},$$

$$G = \{g_1, \dots, g_k\},$$

$$B = \{b_1, \dots, b_k\}.$$

- 2 $\mathcal{F} = \langle G, B \rangle_I$ such that $G(i) = \{g_i\}$ and $B(i) = \{b_i\}$ for $i \in I$.

Q-Rankings

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A **Q-ranking** for \mathcal{FS} is a function $f : Q \rightarrow [1..n] \times I^k$, which is identified with a pair of functions $\langle r, h \rangle$, where $r : Q \rightarrow [1..n]$ is one-to-one and $h : Q \rightarrow I^k$, mapping a state to a permutation of I .

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☞ A Δ -graph is called **Q-ranked** if every level of the graph is associated with a Q ranking.

Q-Graphs

We say that a finite Δ -graph \mathcal{G} whose every level is ranked by the same Q -ranking $f = \langle r, h \rangle$ an Q -word if the following conditions hold.

- For any $q, q' \in Q$, if $r(q) > r(q')$, there exists a path p from $\langle q, 0 \rangle$ to $\langle q', |\mathcal{G}| \rangle$ such that p visits every $B(i)$ for $i \in I$.

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- ☞ For any $q \in Q$, there exists **exactly** k full paths p_1, \dots, p_k from $\langle q, 0 \rangle$ to $\langle q, |\mathcal{G}| \rangle$ such that for any $i \in I$, p_i does not visit $B(h(q)[1]), \dots, B(h(q)[i])$, but visits $B(h(q)[i+1]), \dots, B(h(q)[k])$, and it visits $G(h(q)[i])$, but none of $G(h(q)[j])$ for $j < i$.

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For any Q -ranking f , $(\mathcal{G}_f)^\omega \notin \mathcal{L}(\mathcal{FS})$.

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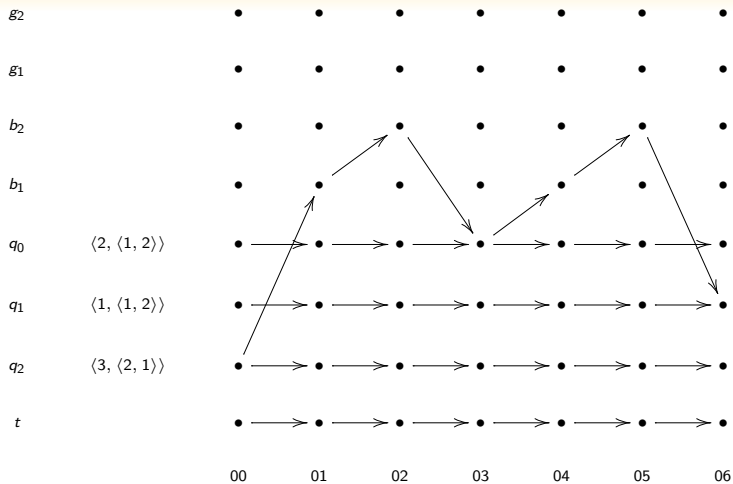
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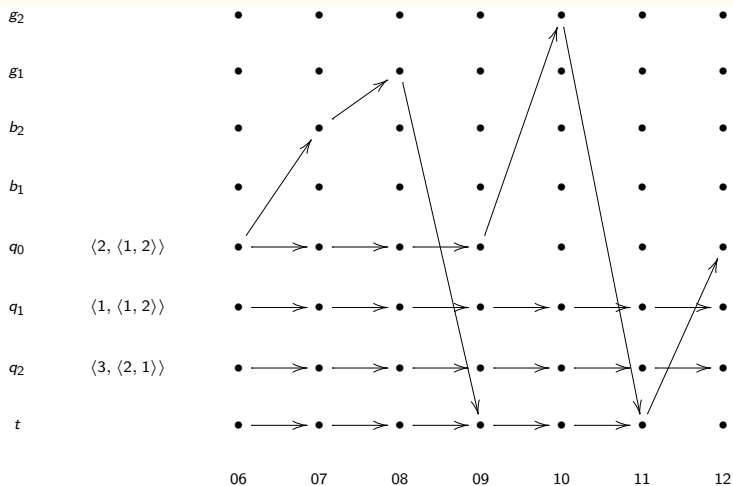
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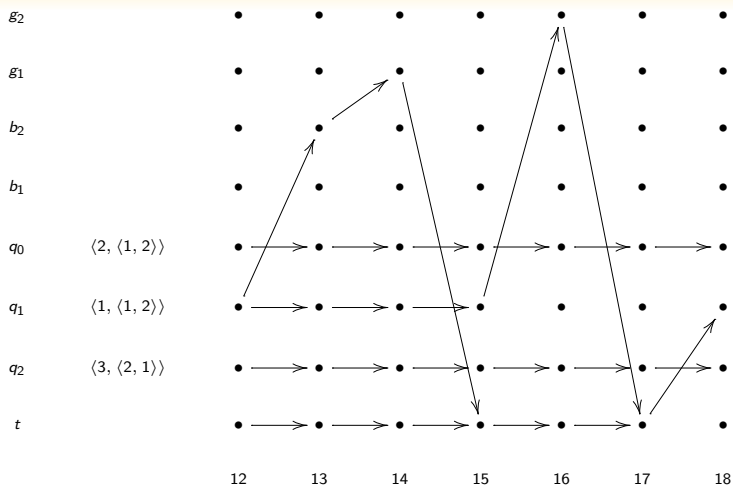
Streett Potential

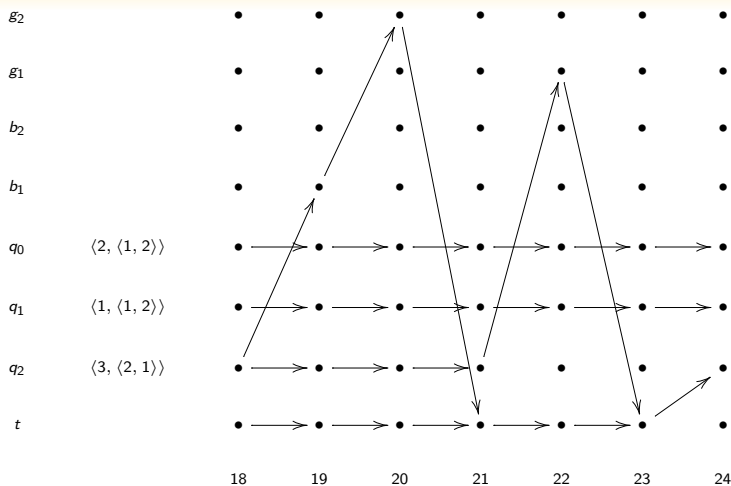
For any two different Q -rankings f and f' , $((\mathcal{G}_f)^+ \circ (\mathcal{G}_{f'})^+)^\omega \subseteq \mathcal{L}(\mathcal{FS})$.

The r -word \mathcal{G}_r



$\mathcal{G}_h^{(1)}$ of the h -word \mathcal{G}_h


$\mathcal{G}_h^{(2)}$ of the h -word \mathcal{G}_h


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Simple Lower Bound

Streett complementation is in $2^{\Omega(N \lg N + KN \lg K)}$ for $K = O(N)$.

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Upper Bound Technique

Ranking Based Complementation

- ➡ An ω -word w is accepted by \mathcal{A} if and only if **all** paths in the Δ -graph \mathcal{G}_w satisfy the dual co- T condition.

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Essential Property

This co- T ranking transforms the **universal property** that “all paths satisfy the co- T condition” into an **existential** property that “some path satisfies a T ” condition.

Generalized Büchi Complementation

Kupferman, Vardi 2005

All paths of a Δ -graph \mathcal{G} satisfy a generalized co-Büchi condition $\langle B \rangle_I$ if and only if there exists a GC-ranking δ on \mathcal{G} such that

- For any edge $\langle v, v' \rangle$, we have $\delta(v) \geq \delta(v')$;
- For any vertex $\langle q, I \rangle$, if $\delta(\langle q, I \rangle) = \langle t, u \rangle$, then $q \notin B(u)$;
- Every path of \mathcal{G} contains infinitely many odd vertices.

Streit Complementmentation

Kupferman, Vardi 2005

- ☞ A Δ -graph \mathcal{G} goes through a finite sequence of graph surgeries, producing a sequence of nonempty graphs $\mathcal{G}_1(= \mathcal{G}), \dots, \mathcal{G}_{\lambda+1}$.

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- ☞ The Rabin ranking of \mathcal{G} is just the tuple $\langle \delta_1, \dots, \delta_{\lambda+1} \rangle$.

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Kupferman, Vardi 2005

The detailed construction goes as follows. At round i , the following four operations are applied to \mathcal{G}_i , producing \mathcal{G}_{i+1} . The procedure repeats until \mathcal{G}_{i+1} is empty.

- (O1) Assign \mathcal{G}_i a GC-ranking δ_i .
- (O2) Remove all vertices v and v 's incoming and outgoing edges if $\delta_i(v)$ is even.
- (O3) Remove all edges $\langle v, v' \rangle$ if $\delta_i(v) > \delta_i(v')$.
- (O4) Remove all edges $\langle v, v' \rangle$ if $\delta_i(v)$ is odd with index (rank) j and v is a $G(j)$ -vertex.

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The detailed construction goes as follows. At round i , the following four operations are applied to \mathcal{G}_i , producing \mathcal{G}_{i+1} . The procedure repeats until \mathcal{G}_{i+1} is empty.

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- (O2) Remove all vertices v and v 's incoming and outgoing edges if $\delta_i(v)$ is even.
- (O3) Remove all edges $\langle v, v' \rangle$ if $\delta_i(v) > \delta_i(v')$.
- (O4) Remove all edges $\langle v, v' \rangle$ if $\delta_i(v)$ is odd with index (rank) j and v is a $G(j)$ -vertex.

Improved Streett Complementation

Our Construction

At round i , the following four operations are applied to \mathcal{G}_i , producing \mathcal{G}_{i+1} . The procedure repeats until \mathcal{G}_{i+1} is empty.

- (P1) Assign \mathcal{G}_i a **joyful** GC-ranking δ_i .
- (P2) Remove all vertices v and v 's incoming and outgoing edges if $\delta_i(v)$ is even.
- (P3) Remove all edges $\langle v, v' \rangle$ if $\delta_i(v) > \delta_i(v')$.
- (P4) Remove all edges $\langle v, v' \rangle$ if $\delta_i(v)$ is odd, $v = \langle q, l \rangle$ and $q \in G(t)$ for some $t \in \text{Cover}(\langle \delta_1, \dots, \delta_i \rangle)[i]$.

Tighter Upper Bounds

Streott Upper Bounds

Streott complementation is in $2^{O(\lg T_{\mathcal{R}} + N \lg T_{\mathcal{H}})}$, and is $2^{O(N^2 \lg N)}$ regardless of how large K is.

Tighter Upper Bounds

Streett Upper Bounds

Streett complementation is in $2^{O(\lg T_{\mathcal{R}} + N \lg T_{\mathcal{H}})}$, and is $2^{O(N^2 \lg N)}$ regardless of how large K is.

Parity Upper Bounds

Parity complementation is in $2^{O(N \lg N + NK)}$ where $K \leq N$.

Outline

- 1 Introduction
- 2 Preliminary
- 3 Lower Bound
- 4 Upper Bound
- 5 Conclusion**

Conclusion

Our Contribution

- ➔ We improve both the lower and upper bounds for Streett complementation to, respectively, $2^{\Omega(\lg T_{\mathcal{R}} + N \lg T_{\mathcal{H}})}$ and $2^{O(\lg T_{\mathcal{R}} + N \lg T_{\mathcal{H}})}$.

Conclusion

Our Contribution

- We improve both the lower and upper bounds for Streett complementation to, respectively, $2^{\Omega(\lg T_{\mathcal{R}} + N \lg T_{\mathcal{H}})}$ and $2^{O(\lg T_{\mathcal{R}} + N \lg T_{\mathcal{H}})}$.
- ☞ We also give a tighter upper bound for parity complementation.

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Future Work

- 👉 Improve lower bounds for other types of automaton transformations

Conclusion

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- We improve both the lower and upper bounds for Streett complementation to, respectively, $2^{\Omega(\lg T_{\mathcal{R}} + N \lg T_{\mathcal{H}})}$ and $2^{O(\lg T_{\mathcal{R}} + N \lg T_{\mathcal{H}})}$.
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Future Work

- Improve lower bounds for other types of automaton transformations
- ☞ In particular, tighten bounds for the complementation of other types of automata

Thank you for your attention!